Integrators & Differentiators
Integrators

- Op Amp RC circuits now
- Opens door to a wide range of useful and exciting applications
- Integrators and Differentiators are two basic applications
Inverting Integrator

Replacing resistors with general impedances

\[
\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}
\]
\[ Z_1 = R_1 \]
\[ Z_2 = R_2 \parallel \frac{1}{sC_2} \]

\( Z_2 \) is the parallel connection of two components.

\( Z_2(s) = \frac{1}{Y_2(s)} \)

\[ \frac{V_o(s)}{V_i(s)} = -\frac{1}{Z_1(s)Y_2(s)} \]

Substitute \( Z_1 = R_1 \) and \( Y_2(s) = \frac{1}{R_2} + sC_2 \)

\[ \frac{V_o(s)}{V_i(s)} = -\frac{1}{\frac{R_1}{R_2} + sC_2R_1} \]

or

\[ \frac{V_o(s)}{V_i(s)} = -\frac{R_2}{R_1} - \frac{R_2}{R_1} + sC_2R_2 \]

Finite dc Gain is given as

\[ K = -\frac{R_2}{R_1} \]

3-dB frequency is

\[ \omega_0 = \frac{1}{C_2R_2} \]

Which is the T.F of a low pass STC network.
• The capacitor behaves as open circuit at dc, so by inspection the dc gain is simply 
  \(-\frac{R_2}{R_1}\)
• Due to the virtual ground at the inverting terminal the resistance seen by the capacitor is simply \(R_2\), hence the time constant of the low pass STC network is \(C_2R_2\)
Output voltage is proportional to the time integral of input voltage and CR is the integrator Time Constant.

Negative sign indicates that it is an Inverting Integrator (also known as Miller Integrator).

\[ i_1(t) = \frac{v_i(t)}{R} \]

\[ v_o(t) = -v_c(t) \]

\[ v_o(t) = -\frac{1}{CR} \int_0^t v_i(t) \, dt - V_C \]
\[
\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}
\]

\[Z_1(s) = R\]

\[Z_2(s) = 1/sC\]

\[
\frac{V_o(s)}{V_i(s)} = -\frac{1}{sCR}
\]

For physical frequencies, \( s = j\omega \)

\[
\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{1}{j\omega CR}
\]

\[
\left|\frac{V_o}{V_i}\right| = \frac{1}{\omega CR}
\]

\[\phi = +90^\circ\]
**Bode Plot For The Inverting Integrator**

- As $\omega$ doubles, the gain is halved (decreases by 6 dB)
- **Intercepts the 0 dB line at $\omega=1/RC$ making gain equal to unity**

This is known as Integrator Frequency

- Behaves as a low pass STC network with a corner frequency of zero
- At $\omega=0$, the T.F becomes infinite as the capacitor is opened and op amp is operating with open loop
- Any tiny dc component in the input will saturate the output which is a serious problem with integrator circuit
Effect of Input Offset DC voltage and current

\[ v_o = V_{OS} + \frac{1}{C} \int_{0}^{t} \frac{V_{OS}}{R} dt \]

\[ = V_{OS} + \frac{V_{OS}}{CR} t \]

\[ v_o = V_{OS} + \frac{1}{C} \int_{0}^{t} \frac{V_{OS}}{R} dt \]

\[ = V_{OS} + \frac{V_{OS}}{CR} t \]

\[ v_o \text{ increases linearly with time until the op amp saturates} \]

\[ \text{the dc input offset current } I_{OS} \text{ produces a similar problem} \]

- \( R \) added in the positive lead to keep the input bias current \( I_B \) from flowing through \( C \)
- Nevertheless \( I_{OS} \) flows through \( C \) and causes the op amp output to saturate.
Alleviation of dc problem

• The dc problem can be alleviated by connecting a resistor \( R_F \) across the integrator capacitor \( C \)
• It provides a dc path for the dc currents \( V_{OS}/R \) and \( I_{OS} \)
• Now \( v_0 \) will have a dc component

\[
\left[ V_{OS} \left( 1 + \frac{R_F}{R} \right) + I_{OS} R_F \right]
\]

instead of rising linearly
• To keep the dc offset low, one would lower \( R_F \)

• Lower the value of \( R_F \), less ideal the integrator circuit becomes
• Inclusion of $R_F$ moves the pole at $\omega=0$ to $\omega=1/CR_F$ and the new T.F becomes

$$\frac{V_o(s)}{V_i(s)} = -\frac{R_F/R}{1 + sCR_F}$$

• Lower the value of $R_F$, higher the corner frequency will be and more non-ideal the integrator becomes

• Selection of $R_F$, a trade off between dc performance and signal performance
The Op Amp Differentiator

- **Interchanging the positions of R and C in an ideal Integrator results in Differentiator circuit**
- **Performs the mathematical operation of signal differentiation**

![Differentiator Circuit Diagram](image)

\[ i(t) = C \frac{dv(t)}{dt} \]
\[ v_O(t) = -CR \frac{dv(t)}{dt} \]
\[ \frac{V_o}{V_i} = -sCR \]

![Frequency Response Graph](image)
\[ i(t) = C \frac{dv_i(t)}{dt} \]
\[ v_O(t) = -CR \frac{dv_i(t)}{dt} \]
\[ \frac{V_o}{V_i} = -sCR \]

for physical frequencies \( s = j\omega \) yields

\[ \frac{V_o(j\omega)}{V_i(j\omega)} = -j\omega CR \]

\[ \left| \frac{V_o}{V_i} \right| = \omega CR \]

\[ \phi = -90^\circ \]

- Bode Plot shows that the magnitude doubles for an octave increase in frequency.
- Plot is a straight line of slope 6dB/octave intersecting the 0 dB line at \( \omega = 1/CR \) (the Differentiator Time Constant).
• The Frequency response of Differentiator can be thought of a High pass STC network with corner frequency at infinity

• Differentiator circuit can act as a Noise Magnifier as a spike appears at the output every time there is a sharp change at the input

• This causes stability problems, so they are generally avoided

• Sometime a small-valued resistor is used in series with the capacitor, but it introduces non-ideal behavior to the circuit