COURSE CURRICULUM
2017-ONWARD

FOR

(i) **MS Mathematics**
(ii) **MS Leading to PhD Mathematics**
(iii) **PhD Mathematics**

DEPARTMENT OF BASIC SCIENCES & HUMANITIES
UNIVERSITY OF ENGINEERING & TECHNOLOGY, TAXILA
AUGUST, 2017

MA-6101 THEORY OF GROUP ACTIONS

Course Title: THEORY OF GROUP ACTIONS

Course Code: MATH 6101

Credit Hours: 3

Prerequisites: Algebra I

Course Contents: Preliminaries, The theory of group actions, Coset spaces, Multiplicative group of finite fields, Extensions of finite fields, projective line over finite fields, Projective and linear groups through actions.

RECOMMENDED BOOKS:

1. Coxeter, H.S.M. and Moser, W.O., Generators and Relations for Discrete Groups, Springer-Verlag, 1972.
2. Rose, S., A Course in Group Theory, Cambridge University Press, 1978.
3. Magnus, W., Karrass, A and Solitar, D., Combinatorial Group Theory, Dover Publications, 1976.
4. Johnson, D.L., Presentation of Groups, Cambridge Lecture Notes, 1976.

MA-6102 APPLIED LINEAR ALGEBRA-I

Course Title: APPLIED LINEAR ALGEBRA-I

Course Code: MATH 6102

Credit Hours: 3

Prerequisites: Linear Algebra, Algebra I

Course Contents: Existence of a linear transformation; Rank and nullity of a linear transformation; The algebra of linear transformations; Representation of transformations by matrices; Isomorphism; Dual Space of a vector space; Algebraic reflexivity; Hyperspace; The annihilator of a set; The transpose of a linear transformation. Concept of diagonalization of a linear transformation. Commutative linear algebra with identity over a field; Polynomial; Algebra of polynomials; Polynomial ideals; Prime factorization of a polynomial; Characteristic and minimal polynomials for a linear operator; Cayley-Hamilton theorem; Multiplicity of a root of a polynomial; Invariant subspaces; Simultaneous Triangulation and simultaneous diagonalization; Direct–sum decompositions; Invariant direct sums; The primary decomposition theorem.

RECOMMENDED BOOKS:

1. Apostol, T.M., “Linear Algebra”, Wiley International, 1997.
2. Hoffman, K.M. and Kunze, R.,“Linear Algebra”, Prentice-Hall, 1971.
3. Karamat H. Dar, “Linear Algebra”, Carvan Book House-Lahore, 2007.
4. Lipschutz, S., “Schaum’s Outline of Linear Algebra”, McGraw Hill, 2002.
5. Mey, C.D., “Matrix Algebra and Applied Linear Algebra”, SIAM Publications, 2001.
6. Strang, G., “Introduction to Linear Algebra”, Wellesley-Cambridge Press, 1998.

MA-6103 THEORY OF GROUP GRAPHS

Course Title: THEORY OF GROUP GRAPHS

Course Code: MATH 6103

Credit Hours: 3

Prerequisites: Algebra I

Course Contents: Graphs, graphs for group actions, projective special linear group and its action on real, rational and irrational fields, Graphical representations of Mobius, Orthogonal, Affine and Euclidean groups.

RECOMMENDED BOOKS:

1. Coxeter, H.S.M. and Moser, W.O., Generators and relations for discrete groups, Springer-Verlag, 1972.
2. Rose, S., A course in group theory, Cambridge University Press, 1978.
3. Magnus, W., Karrass, A and Solitar, D., Combinatorial group theory, Dover Publications, 1972.

MA-6104 ADVANECD MATHEMATICAL MODELLING

Course Title: ADVANECD MATHEMATICAL MODELLING

Course Code: MATH 6104

Credit Hours: 3

Prerequisites: Linear Algebra, Calculus, Discrete Mathematics

Course Contents: Ordinary differential equations, Modelling with first order differential equations, Population growth model, Competitive Hunter model , Predator Prey model, Introduction to compartment model, Modelling with 2nd order differential equations, Application to biological system, Modelling with wave, heat, and Laplace equations.Partial differential equations methodology of mathematical modelling, Traffic flow, RC Circuits, Introduction to delay differential equations, modelling technique in case of delay differential equations(Cancer model, Baroreceptor model)

RECOMMENDED BOOKS:

1. Mathematical Modelling by JagatNarainKapur Edition 1998.
2. Cardiovascular and Respiratory Systems: Modeling, Analysis, and Control by Jerry J. Batzel, Franz Kappel, Daniel Schneditz, Hien T. Tran Edition October 2005**.**
3. Cardiovascular Mathematics: Modeling and simulation of the circulatory system by von Luca Formaggia,AlfioQuarteroni,AlessandroVeneziani.
4. Applied Mathematical Models in Human Physiology byJohnny T. Ottesen , Mette S. Olufsen and Jesper K. Larsen.

MA-6105 VISCOUS FLUID FLOW

Course Title: VISCOUS FLUID FLOW

Course Code: MATH 6105

Credit Hours: 3

Prerequisites: Linear Algebra, Calculus, Discrete Mathematics

Course Contents: Stress-strain rate relationships for a Newtonian Fluid, Navier-Stokes' equations, Exact solutions of Navier-Stokes' equations, Dimensional analysis, Dynamical similarity, Concepts of themodynamics, Energy equation, Exact solutions for the problem of temperature distribution in a viscous flow, boundary layer concept and its governing equations. Exact solutions of boundary layer equations, Von Karman momentum integral equations and its application, Energy integral equation, Approximate solutions of steady boundary layer equations: Von Karman-pohlhausen method, Walz method, Thwaite's method, Introduction to turbulent boundary layer flow.

RECOMMENDED BOOKS:

1. Schilchting, H., "Boundary Layer Theory (Seventh edition)" McGraw-Hill, 1979.
2. White, F. M., "Viscous Fluid Flow (Second edition)" McGraw-Hill, 1991.
3. Churchill, S.W., "Viscous Flows" Butterworth Publishers, 1988.
4. Hughes, W.F., "An Introduction to Viscous Flow" Hemisphere, N.Y.,1979.
5. Yaun, S.W., "Foundations of Fluid Mechanics" Prentice hall, 1967.

MA-6106 PARTIAL DIFFERENTIAL EQUATIONS

Course Title: PARTIAL DIFFERENTIAL EQUATIONS

Course Code: MATH 6106

Credit Hours: 3

Prerequisites: ODE's,

Course Contents: Cauchy's Problems for Linear Second Order Equations in n-Independent Variables. Cauchy Kowalski Theorem. Characteristic surfaces. Adjoint operations, Bi-characteristics. Spherical and Cylindrical Waves. Heat equation, Wave equation, Laplace equation, Maximum-Minimum Principle, Integral Transforms.

RECOMMENDED BOOKS:

1. Dennemyer, R., Introduction to Partial Differential Equations and Boundary Value Problems, McGraw-Hill Book Company, 1968.
2. Chester, C.R., Techniques in Partial Differential Equations, McGraw-Hill Book Company, 1971.

MA-6107 MATHEMATICAL TECHNIQUES FOR BOUNDARY VALUE PROBLEMS

Course Title: MATHEMATICAL TECHNIQUES FOR BOUNDARY VALUE PROBLEMS

Course Code: MATH 6107

Credit Hours: 3

Prerequisites: ODE's,

Course Contents: Green's function method with applications to wave-propagation. **Perturbation method:** regular and singular perturbation techniques with applications, Variational methods,A survey of transform techniques, Wiener-Hopf technique with applications to diffraction problems.

RECOMMENDED BOOKS:

1. Nayfeh, A., Perturbation methods, 2008.
2. Stakgold, I., Boundary value problems of Mathematical Physics, 1968.
3. Noble, B., Methods based on the Wiener-Hopf technique for the solution of Partial Differential Equations, 1958.
4. Mitra, R., and Lee, S.W., Analytical Techniques in the Theory of Guided Waves, MacmillanComp.,New York 1971.

MA-6108 FUZZY ALGEBRA

Course Title: FUZZY ALGEBRA

Course Code: MATH 6108

Credit Hours: 3

Prerequisites: Algebra

Course Contents: Introduction, The Concept of Fuzziness: Examples; Mathematical Modeling; Operations of fuzzy sets; Fuzziness as uncertainty. Algebra of Fuzzy Sets: Boolean Algebra and lattices; Equivalence relations and partitions; Composing mappings; Alpha-cuts; Images of alpha-level sets; Operations on fuzzy sets. Fuzzy Relations: Definition and examples; Binary Fuzzy relations Operations on Fuzzy relations; fuzzy partitions. Fuzzy Semigroups, Fuzzy ideals of semigroups; Fuzzy quasi-ideals; Fuzzy bi-ideals of Semigroups; Characterization of different classes of semigroups by the properties of their fuzzy ideals fuzzy quasi-ideals and fuzzy bi-ideals. Fuzzy Rings: Fuzzy ideals of rings; Prime; semiprime fuzzy ideals; Characterization of rings using the properties of fuzzy ideals.

RECOMMENDED BOOKS:

1. Hung T. Nguyen and Elbert A. Walker, A First Course in Fuzzy Logic, , Chapman and Hall/CRC 1999.
2. M. Ganesh,Introduction to Fuzzy Sets and Fuzzy Logic, Prentice-Hall of India, 2006.
3. John N. Mordeson and D. S. Malik, Fuzzy Commutative algebra, World Scientific, 1998.
4. John N. Mordeson, D. S. Malik and Fuzzy Semigroups, Springer-Verlage, Nobuki Kuroki, 2003.

MA-6109 APPLIED FUNCTIONAL ANALYSIS-I

Course Title: APPLIED FUNCTIONAL ANALYSIS-I

Course Code: MATH 6109

Credit Hours: 3

Prerequisites: Algebra

Course Contents: Bounded linear operators, Bounded linear functionals, Riesz theorem for functionals on Hilbert spaces, Riesz representation theorem, Existence theorem for Hilbert-adjoint operator, Properties of Hilbert-adjointoperators, Sequences of self-adjoint operators, Unitary and normal operators. Hahn Banach theorems, Norm of the adjoint operator, Relation between the adjoint operator and the Hilbert-adjoint operator, Reflexive spaces, Baire's category theorem; Uniform boundedness theorem; Open mapping theorem; Closed graph theorem; Banach fixed point theorem; Properties of Banach algebras.

RECOMMENDED BOOKS:

1. F. Riesz and Nagy, "Functional Analysis", Frederick unger publishing Co, 1965.

2. E. Kreyszig, “Introductory Functional Analysis with Applications”, John, Wiley and Sons, New York, 1989.
3. A.E. Taylor, “Introduction to Functional Analysis”, wiley International Edition, New York, 1957.
4. W. Rudin, “Functional Analysis”, McGraw-Hill, Inc., New York, 1991.
5. M.T. Nair, “Functional Analysis”, Prentice Hall of India, New Delhi, 2002.

MA-6110 COMPRESSIBLE FLUID FLOW

Course Title: COMPRESSIBLE FLUID FLOW

Course Code: MATH 6110

Credit Hours: 3

Prerequisites: ODE’s, PDE’s

Course Contents: Concepts of thermodynamics, Mach Number, Types of flows, Inviscid compressible flow, Continuity, Euler’s, Bernoulli’s, and Energy Equations, kelvin’s theorem, Potential function equation, Stream function, Shock waves, method of characteristics, Linearisation and small perturbation methods, Viscous compressible flow, Navier Stokes’ equations for a compressible viscous flow, Exact solutions, Relation between velocity and temperature fields with and without pressure gradient, Energy equation laminar, boundary layer equation, Velocity and temperature relation in laminar boundary layer, Momentum and energy integral equations.

RECOMMENDED BOOKS:

1. Misos, R.V., “Mathematical Theory of Compressible Fluid Flow” Academic Press, 1966.
2. Chapman, A.J. and W.F. Walker, “Introductory Gas Dynamics” Holt N.Y, 1976.
3. Saad, M.A., “Compressible Fluid Flow”, Prentice-Hall, 1985.
4. Schreier, S., “Compressible Flow:, John-Wiley & Sons, 1982.
5. Thompson, P.A., “Compressible Fluid Dynamics”, McGraw-Hill, 1972.

MA–6111 INTEGRAL TRANSFORMS

Course Title: INTEGRAL TRANSFORMS

Course Code: MATH 6111

Credit Hours: 3

Prerequisites: Linear Algebra, Calculus

Course Contents: Introduction, Fourier Transforms, The Mellin Transforms, The Henkel Transforms, The Kanotorovich – Lebeder Transforms, The MehlerForch transform, Finite transforms, Generalised Functions.

RECOMMENDED BOOKS

1. Sneddon, I.N., "The use of Integral Transforms", McGraw-Hill, 1972.
2. Davies, B., "Integral Transforms and their Applications" , Springer-Verlag, 2002.
3. Gelfand, I.M. and Vilenkin, N. Ya., "Generalised Functions Vol. I, II" Academic Press, 1977.
4. Debnath, L., "Intergral Transforms and their Applications" CRC Press, 1995.
5. Titchmarsh, I., "The Theory of Functions", Oxford university Press, 1970.

MA-6112 MAGNETOHYDRODYNAMICS-I

Course Title: MAGNETOHYDRODYNAMICS-I

Course Code: MATH 6112

Credit Hours: 3

Prerequisites: Linear Algebra, Calculus

Course Contents: Basic Equations: Equations of electrodynamics, Equations of Fluid Dynamics, Ohm's law equations of magnetohydrodynamics.

Motion of an Incompressible Fluid: Motion of a viscous electrically conducting fluid with linear current flow, steady state motion along a magnetic field, wave motion of an ideal fluid.

Small Amplitude MHD Waves: Magneto-sonic waves. Alfve's waves, damping and excitation of MHD waves, characteristics lines and surfaces.

Simples Waves and Shock Waves in Magnetohydrodynamics: Kinds of simple waves, distortion of the profile of a simple wave, discontinuities, simple and shock waves in relativistic magnetohydrodynamics, stability and structure of shock waves, discontinuities in various quantities, piston problem, oblique shock waves.

RECOMMENDED BOOKS:

1. Cowling, T.G., Magnetohydrodynamics, Interscience Publishers, 1963.
2. Kulikowsky, A.G., and Lyabimov, A.G., Magnetohydrodynamics, A.Wesley, 1965.
3. Alfve's H., and Falthammar, C., Cosmical Electrodynamics, Clarendon Press, 1965.
4. Akhiezer et.al. Plasma Electrodynamics, Pergamon Press, 1975.

5. Kendale and Plumpton, C. Magnetohydrodynamics.
6. Anderson, J.E. Magnetohydrodynamics, Shock Waves, M.I.T. Press, 1975.

MA-6113 MAGNETOHYDRODYNAMICS-II

Course Title: MAGNETOHYDRODYNAMICS-II

Course Code: MATH 6113

Credit Hours: 3

Prerequisites: Linear Algebra, Calculus
Course Contents:

Flow of Conducting Fluid Past Magnetized Bodies: Flow of an ideal fluid past magnetized bodies, Fluid of finite electrical conductivity flow past a magnetized body.

Dynamo Theories: Elsasser's Theory, Bullard's Theory, Earth's field Turbulent motion and dissipation, vorticityanology.

Ionized Gases: Effects of molecular structure, Currents in a fully ionized gas, partially ionized gases, interstellar fields, dissipation in hot and cool clouds.

RECOMMENDED BOOKS:

1. Gowling, T.G., Magnetohydrodynamics, Interscience Publishers, 1963.
2. Kulikowskiy, A.G., and Lyubimov, G.A., Magnetohydrodynamics, A.Wesley, 1965.
3. Alfven, H., and Falthammar, C., Cosmical Electrodynamics, Clarendon Press, 1965.
4. Akliezer, A.I., Plasma Electrodynamics, Pergamon Press, 1975.
5. Kendall, P.C., and Plumption, C., Magnetohydrodynamics.
6. Aderson, J.E. Magnetohydrodynamics Shock Waves, M.I.T. Press, Cambridge, 1963.

MA-6114 INTEGRAL EQUATIONS

Course Title: INTEGRAL EQUATIONS

Course Code: MATH 6114

Credit Hours: 3

Prerequisites: Linear Algebra, Calculus

Course Contents: Existence Theorems, Integral Equations with L2 Kernels. Applications to partial differential equations. Integral transforms(Fourier Transforms, The Mellin Transforms, The Henkel Transforms, The Kanotorovich – Lebeder Transforms, The Mehler Forch transform, Finite transforms), Wiener-Hopf Techniques, Generalized functions.

RECOMMENDED BOOKS:

1. Harry Hoch Stadl, Integral Equations, John Wiley, 1973.

2. Stakgold, I., Boundary Value Problems of Mathematical Physics, Macmillan, New York, 1968.

MA-6115 NON-NEWTONIAN FLUID MECHANICS

Course Title: NON-NEWTONIAN FLUID MECHANICS

Course Code: MATH 6115

Credit Hours: 3

Prerequisites: Linear Algebra, Calculus

Course Contents: Classification of Non-Newtonian Fluids, Rheological formulae (Time-independent fluids, Thixotropic fluids and viscoelastic fluids), Variable viscosity fluids, Cross viscosity fluids, The deformation rate, Viscoelastic equation, Materials with short memories, Time dependent viscosity. The Rivlin-Ericksen fluid, Basic equations of motion in rheological models. The linear viscoelastic liquid, Couette flow, Poiseuille flows. The current semi-infinite field, Axial oscillatory tube flow, Angular oscillatory motion, Periodic transients, Basic equations in boundary layer theory, Orders of magnitude, Truncated solutions for viscoelastic flow, Similarity solutions, Turbulent boundary layers, Stability analysis.

RECOMMENDED BOOKS:

1. John Harris, Theology and Non-Newtonian Flow, Longman Inc, New York 1977.
2. W.R. Schowalter, Mechanics of Non-Newtonian fluids, New York, Pergamon Press 1978.
3. R.B. Birk, R.c. Armstrong and O. Hassager, Dynamics of Polymeric liquids, Vol. 1, 2nd ed., John Wiley & Sons, New York, NY 1987.
4. G Astarita and G. Merrucci. Principles of Non-Newtonian Fluid mechanics, McGraw-Hill 1974.

MA-6116 NUMERICAL SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS

Course Title: NUMERICAL SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS Course Code: MATH 6116

Credit Hours: 3

Prerequisites: Linear Algebra, Numerical Analysis

Course Contents:

1. Boundary and intial conditions, Polynomial approximations in higher dimensions.
2. **Finite Element Method:** The Galerkin method in one and more dimensions, Error bound on the Galarki method, The method of collocation, Error bounds on the

collocation method, Comparison of efficiency of the finite difference and finite element method.

3. **Finite Difference Method:** Finite difference approximations.
4. Application to solution of linear and non-linear Partial Differential Equations appearing in Physical Problems.

RECOMMENDED BOOKS:

1. Strang G., and Fix G., An Analysis of Fintie Element Method, Prentice Hall, New Jersey 1973.
2. David S. Burnett., Finite Element Analysis from Concepts to Applications, Addision Wesley, 1987.
3. Myron B. Allen., Ismael Herrera and George F., Pinder Numerical Modeling in Science and Engineering, John Willy & Sons, New York, 1988.
4. Desai, G.S., Elementary Finite Element Method, Prentice Hall, Inc. 1988.

MA-6117 PERTURBATION METHODS IN FLUID MECHANICS

Course Title: PERTURBATION METHODS IN FLUID MECHANICS

Course Code: MATH 6117

Credit Hours: 3

Prerequisites: Fluid Mechanics, Numerical Analysis

Course Contents: Limit procedures, Order notation, Definition, Matching principle, simple algebraic and differential equations, Thin aerofoil theory, Supersonic aerofoil theory, Inviscid shallow water waves, P-L-K; Vander Pol equation, Non-linear oscillations, boundary layers flow, Low Reynolds number flow, Lubrication theory, Steepest descents and second order ordinary differential equations.

RECOMMENDED BOOKS

1. Dyke, M.V. "Perturbation Methods in fluid Mechanics", Parabolic Press, 1975.
2. Cole, J.D. and Kevorkian, J., "Perturbation Methods in Applied Mathematics", Springer Verlag, 1985.
3. Nayfeh, A.H., "Perturbation Methods", John Wiley and sons, 2000.
4. Wasow, W.R., "Asymptotic Expansions for O.D.E.", R.E. Dreiger Publishing Company, 1985.
5. Copson, E.T., "Asymptotic Expansions", SIAM Publications, 1980.

MA-6118 NILPOTENT AND SOLUBLE GROUPS

Course Title: NILPOTENT AND SOLUBLE GROUPS

Course Code: MATH 6118

Credit Hours: 3

Prerequisites: Algebra, group Theory

Course Contents: Normal and Subnormal Series, Abelian and Central Series, Direct Products, Finitely Generated Abelian Groups, Splitting Theorems, Solube and Nilpotent Groups, Commutators Subgroup, Derived Series, The Lower and Upper Central Series, Characterization of Finite Nilpotent Groups, Fitting Subgroup, Frattini Subgroup, Dedekind Groups, Supersoluble Groups, Solube Groups with Minimal Condition. Subnormal Subgroups, Minimal Condition on Subnormal Subgroups, The Subnormal Socle, the Wielandt Subgroup and Wielandt Series, T-Groups, Power Automorphisms, Structure and Construction of Finite Soluble T-Groups.

RECOMMENDED BOOKS:

1. Robinson, D.J.S., A Course in the Theory of Groups, Graduate Textes in Mathematics 80, Springer, New York, 1982.

2. Doerk, K. and Hawkes, T., Finite Soluble Groups, De Gruyter Expositions in Mathematics 4, Walter De Gruyter, Berlin, 1992.

MA-6119 THEORY OF SPLINES-I

Course Title: THEORY OF SPLINES-I

Course Code: MATH 6119

Credit Hours: 3

Prerequisites: ODE's, Linear Algebra

Course Contents: Spline function of one variable: Interpolating cabic splines, smoothing cubic splines. Spline functions of two variables: Interpolentingbicubic splines, smoothing bicubic splines. Geometric splines: Spline curres, Be'zier Curves, B-spline curves. Spline surfaces: Be'zier surfaces, B-spline surfaces.

RECOMMENDED BOOKS:

1. Eugene V. Shikin& Alexander-I Plis, " Handbook on splines", CRC Press, 1995.
2. Larry L. Sehumaker, "Spline Functional: Basic Theory", 3rd ed, Cambridge University Press, 2007.
3. Carl de Boor, "A Practical Guide to Splines" Springer, 2000.

4. J.H. Ahlberg& E.N Nilson, “The Theory of Splines & their applications” Academic Press, 1967.
5. Charles K. Chui, “Multinariable Splines”, SIAM Publications, 1991.

MA-6120 LA-SEMIGROUPS

Course Title: LA-SEMIGROUPS

Course Code: MATH 6120

Credit Hours: 3

Prerequisites: Group Theory, Linear Algebra

Course Contents: LA-semigroups and basic results, Connection with other algebraic structures, Medial and exponential properteis, LA-semigroups defined by commutative inverse semigroups, Homomorphism theorems for LA-semigroups, Abelian groups defined by LA-semigroups, Embedding theorem for LA-semigroups, Structural properties of LA-semigroups, LA-semigroups as a semilattice of LA-subsemigroups, Locally associative LA-semigroups, Relations on locally associative LA-semigroups, Maximal separativehomomorphic images of locally associative LA-semigroups, Decomposition of locally associative LA-semigroups.

RECOMMENDED BOOKS:

1. Clifford, A.H. and G.B. Preston., The Algebraic Theory of Semigroups, Vols. I & II, Amer. Math. Soc. Surveys, 7, Providence, R.I, 1967.

MA-6121 NUMERICAL SOLUTIONS OF NON-LINEAR SYSTEM OF EQUATIONS AND ORDINARY DIFFERENTIAL EQUATIONS

Course Title: NUMERICAL SOLUTIONS OF NON-LINEAR SYSTEM OF EQUATIONS AND ORDINARY DIFFERENTIAL EQUATIONS

Course Code: MATH 6121

Credit Hours: 3

Prerequisites: Linear Algebra, Numerical Analysis

Course Contents: Iterative techniques in matrix algebra. Estimates and iterative refinerment. Numerical solutions of non linear system of equations. Fixed points for functions of several variables. Newton’s method for systems. Error estimates for fixed point and Newton’s method. Differential & Difference Equations: Differential Equation Problems Differential Equation Theory, Difference Equation Problems, Difference Equation Theory.

Numerical Differential Equations methods: Euler's method, Analysis of Euler Method, Generalizations of Euler's method, Runge-Kutta methods: Order conditions, Runge-Kutta methods with Error Estimates, Stability of Implicit Runge-Kutta methods. Linear Multistep methods: Order of Linear Multistep Methods, Error and Error Growth, Order & Stability Barriers.General Linear Multistep Methods: Consistancy, Stability and convergence, Stability of General Linear Methods, Hybrid Methods.

RECOMMENDED BOOKS:

1. J.C. Butcher, "Numerical Methods for Ordinary Differential Equations", John Wiley & Sons, 2008.
2. Burben, R. L. and Faires, J. D., (6^{th} Ed.), "Numerical Analysis", Brooks Cole Publishing Company, 1997.
3. Leon Lapidus& John H. Seinfeld," Numerical Solutions of Ordinary Differential Equations", Academic Press, 1971.
4. J.D. Lambort, "Numerical Methods for Ordinary Differential Systems", Wiley, 1991.
5. AriehIserles, "A First Course in the Numerical Analysis of Differential Equations", Cambridge University Press, 1996.

MA-6122 NUMERICAL SOLUTIONS OF INTEGRAL EQUATIONS

Course Title: NUMERICAL SOLUTIONS OF INTEGRAL EQUATIONS

Course Code: MATH 6122

Credit Hours: 3

Prerequisites: Integral Equations, Numerical Analysis

Course Contents: Linear integral equations of first & second kinds, solution of integral equations of seconds kind by successive substitutions. The Fredholm theory and its applications, Hilbert, Schmidt Theory of Integral Equations.

RECOMMENDED BOOKS:

1. Lovitt, W.V., "Linear Integral Equations", Dover Publications, 1950.

2. Burden, R.L. and Fairs, J.D., "Numerical Analysis", (Sixth edition), Brooks/Cole Publishing Co, 1997.

3. Mikhlin, S.G., "Integral Equations", Taylor & Francis, 1961.

4. Tricomic, F.G., “Integral Equations”, Dover Publications, 1985.

5. Mathews, J.H., “Numerical Methods for mathematics, science and Engineering”, (Second edition), Prentice Hall, 1992.

MA-6123 APPLIED LINEAR ALGEBRA-II

Course Title: APPLIED LINEAR ALGEBRA-II

Course Code: MATH 6123

Credit Hours: 3

Prerequisites: Linear Algebra

Course Contents: Adjoint operators; Unitary operators; Normal operators; Sesqui-linear form; Principal axis theorem; Positive forms; Properties of normal operators; Symmetric bilinear forms; Skew-symmetric bilinear forms; Orthogonal group; Pseudo-orthogonal group;Cyclic subspaces and annihilators; Companion matrix; Cyclic decomposition theorem; Rational form of a matrix; Jordan canonical form of a matrix; Hermitian forms and the spectral theorem; Spectral theorem for normal operators.

RECOMMENDED BOOKS:

1. Apostol, T.M., “Linear Algebra”, Wiley International, 1997.
2. Hoffman, K.M. and Kunze, R., (2^{nd} Ed.), “Linear Algebra”, Prentice-Hall, 1971.
3. Karamat H. Dar,. “Linear Algebra”, Carvan Book House-Lahore, 2007.
4. Lipschutz, S., “Schaum’s Outline of Linear Algebra”, McGraw Hill, 2002.
5. Mey, c.D., “Matrix Algebra and Applied Linear Algebra”, SIAM Publications, 2001.
6. Strang, G., “Introduction to Linear Algebra”, Wellesley-Cambridge Press, 1998.

MA-6124 APPLIED FUNCTIONAL ANALYSIS-II

Course Title: APPLIED FUNCTIONAL ANALYSIS-II

Course Code: MATH 6124

Credit Hours: 3

Prerequisites: Linear Algebra, real Analysis

Course Contents: Applications to bounded linear functionals to the space of real-valued continuous functions defined on $[a, b]$; Weak convergence; Weak completeness; Convergence of sequences of operators and functionals; Toeplitz limit theorem; Cesaro’ssummability method; Holder’s summability method; Euler’s method. Weierstrass approximation theorem; Polya convergence theorem; Steklov’s theorem; Error estimates;

Hellinger-Toeplitz theorem. Application of Banach’s theorem to linear equations, differential equations and integral equations; Unbounded linear operators in quantum mechanics.

RECOMMENDED BOOKS:

1 F.Riesz and Nagy, “Functional Analysis”, Frederick unger publishing Co, 1965.

2. E. Kreyszig, “Introductory Functional Analysis with Applications”, John, Wiley and Sons, NewYork, 1989.
3. A.E. Taylor. , “Introduction to Functional Analysis”, wiley International Edition, New York, 1957.
4. W. Rudin., “Functional Analysis”, McGraw-Hill, Inc., New York, 1991.
5. M.T. Nair., “Functional Analysis”, Prentice Hall of India, New Delhi, 2002.

MA-6125 ADVANCED COMPLEX ANALYSIS

Course Title: ADVANCED COMPLEX ANALYSIS

Course Code: MATH 6125

Credit Hours: 3

Prerequisites: Complex Analysis

Course Contents: Analytic functions, Analytic functions as mappings, The open mapping theorem, The maximum principle, Schwartz lemma, Convex functions and Hadamard’s three-circle theorem, Elementary properties of holomorphic functions, Harmonic functions, Poisson’s integral formula and Drichlet’s problem, Conformal mapping theorem, Analytic continuation, Monodromy theorem, Riemann surfaces, Modular functions and Picard theorem, Product theorems, Elliptic functions, Non-isolated removability theorems, Prime number theorem.

RECOMMENDED BOOKS:

1. Lars Ahlfors., “Complex Analysis: An Introduction to the Theory of Analytic Functions of one Complex Variable”, McGraw-Hill Co, 1979.
2. John B. Conway., “Functions of One Complex Variables I & II”, Springer, 1995.
3. Serge Lang., “Complex Analysis”, Springer, 2008.
4. Jane P. Gilman and Irwin Kra and Rubi E., “Complex Analysis: In the Spirit of Lipman Bers”,
5. Theodore Gamelin., “Complex Analysis”, Springer, 2001.

MA-6126 ADVANCED OPERATIONS RESEARCH-I

Course Title: ADVANCED OPERATIONS RESEARCH-I

Course Code: MATH 6126

Credit Hours: 3

Prerequisites: Linear Algebra

Course Contents: Simplex algorithm and sensitivity analysis, Dual simplex method, Interior-Point Algorithm, Shortest-path problem, Integer programming, Graphical illustration of nonlinear programming problems One-dimensional unconstraint optimization, Multivariable unconstraint optimization, Steepest Descent method, Newton's method, The Karush-Kuhn-Tucker conditions, Quadratic programming.

RECOMMENDED BOOKS:

1. Hillier, F.and Lieberman, G.J., "Introduction to Operations Research", McGraw Hill,(7th Ed.), 2001.
2. Taha, H.A., "Operations Research: An Introduction", McGraw Hill, (7th Ed.), 2002.
3. Bertsimas, D. and Tsitsikhis, J.N., "Introduction to Linear Optimization", Athena Scientific Publications, 1997.
4. Sofer, A. and Nash, S.G., "Linear and Non Linear programming", McGraw Hill, 1995.
5. Radin, R.L., "Optimization in Operations Research", Prentice Hall, 1997.

MA-6127 ADVANCED OPERATIONS RESEARCH - II

Course Title: ADVANCED OPERATIONS RESEARCH-II

Course Code: MATH 6127

Credit Hours: 3

Prerequisites: Linear Algebra

Course Contents: Single variable optimisation: examples and methods. Examples of optimisations in n-variable. n-variable unconstrained optimisation. Direct search methods: univariate, Nelder& Mead simplex. Line search methods: Steepest Descent, Wolfe's, Armijo. Derivative methods: Newton and Quasi-Newton. Equality constrained optimisation. Penalty Function methods. Inequality constrained optimisation. Interior point and Barrier methods.

RECOMMENDED BOOKS

1. Dimitri P. Bertsekas, "Nonlinear Programming", 2nd ed., Athena Scientific, 1999.
2. Singiresu S. Rao, "Engineering Optimization", 4thed, John Wiley & Sons, 2009.
3. Michael Bartholomew-Biggs, "Nonlinear Optimization with Engineering Applications", Springer, 2008.
4. Jorge Nocedal& Stephen J. Wright, "Numerical Optimization", 2nd Ed., Springer, 2006.
5. Gionni Di Pillo& Fabio Schoen, "Nonlinear Optimization", Springer, 2007.

MA-6128 SEMIGROUP THEORY

Course Title: SEMIGROUP THEORY

Course Code: MATH 6128

Credit Hours: 3

Prerequisites: group Theory

Course Contents: Introductory ideas: Basic definitions, Cyclic semigroups; Ordered sets, semi lattices and lattices. Binary relations; Equivalences; Congruences; Free semigroups; Green's Equivalances; L,R,H,J and D; Regular semigroups, O-Simple semigroups; Simple and O-Simple semigroups; Rees's theorem; Primativeidempotents; Completely O-Simple semigroups; Finite congruence-free semigroups, Union of groups; Bands; Free bands; varieties of bands, Inverse semigroups, Congruences on inverse semigroups; Fundamental inverse semigroups; Bisimple and simple inverse semigroups. Orthodox semigroups; Basic properties; The structure of orthodox semigroups.

RECOMMENDED BOOKS:

1. A.H. Clifford and G.B. Preston, The Algebraic Theory of Semigroups; Vol.I& II. AMS Math. Surveys, 1961 and 1967.
2. J.M. Howie, An Introduction to Semigroup Theory, Academic Press 1976.

MA-6129 THEORY OF ORDINARY DIFFERNTIAL EQUATIONS

Course Title: THEORY OF ORDINARY DIFFERNTIAL EQUATIONS

Course Code: MATH 6129

Credit Hours: 3

Prerequisites: Linear Algebra, ODE's
Course Contents:

Systems of Linear Differential Equations:

Linear systems, Solution matrix, Fundamental solution matrix, Eigen values, Different methods for solving the homogeneous systems, Autonomous systems, Critical points and periodic solutions, Stability of linear systems, Linearization and local stability, Applications of autonomous systems.

Partial Differential Equations:

Classification of equations, Linear and quasi linear first order equations, Method of Lagrange, Cauchy problem for first order and higher order equations in n-independent variables, Normal forms, Hyperbolic, Parabolic, and Elliptic equations.

RECOMMENDED BOOKS

1. Dennis G. Zill and Michwel R. Cullen, “Differential Equations with Boundary Value Problems”, PWS Publishing Co., Boston, USA, (3rd Ed.), 1993.
2. P. A. Glending, “Stability, Instability and Chaos”, Cambridge University Press, 1994.
3. Richard E. William Son, “Introduction to Differential Equations and Dynamical Systems”, McGraw-Hill International Ed, 1997.
4. Rene Dennemeyer, “Introduction to Partial Differential Equations Boundary Value Problems”, McGraw-Hill Book Co, 1968.
5. George F. Carrier and Carl E. Pearson, “Partial Differential Equaitons”, Academic Press, New York, 1976.

MA-6130 GRAPH THEORY

Course Title: GRAPH THEORY

Course Code: MATH 6130

Credit Hours: 3

Prerequisites: Linear Algebra

Course Contents: Fundamentals. Definition. Three puzzles, Paths, cycles and trees. Hamilton cycles and Euler circuits.Planer graphs. Flows, Connectivity and Matching Network flows. Bipartite graphs and their properties.Connectivity and Menger’s theorem. External problems paths and Complete Subgraphs. Hamilton path and cycles. Colouring. Vertex colouring Edge colouring. Brook`s theorem, Chromatic polynomials. Graph on surfaces, Dual graphs, infinite graphs.Digraphs. Eulerian digraphs and tournaments. Marker chains. Hall`s marriage theorem and its applications. Transversal theory. Introduction to Matroids, Matroids and graphs. Matroids and transverals

RECOMMENDED BOOKS:

1. Bollobas, B.: Graph Theory (Springer Verlag, NewYork, 1979).
2. Wilson, R.J.: Introduction to Graph Theory (Longman London, 1979).
3. Bollobas, B.: Modern Graph Theory (Springer Verlag, NY, 2002).
4. Biggs, N.: Algebraic Graph Theory (Cambridge University Press, 1974).
5. Gross, J.L and Yellen, J.: Graph Theory and its Applications (Chapman and Hall, 2005).

MA-6131 PROBABILITY AND RANDOM PROCESS

Course Title: PROBABILITY AND RANDOM PROCESS

Course Code: MATH 6131

Credit Hours: 3

Prerequisites: Probability & Statistics

Course Contents: Probability space and axioms, basic laws, conditional probability and Bayes rule, independence. Random variables, probability mass function (pmf), cumulative distribution function (cdf), probability density function (pdf), discrete, continuous, and mixed random variables, functions of random variables, generation of random variables.Pairs of random variables, joint, marginal, and conditional distributions, maximum likelihood (ML) and maximum a posteriori probability (MAP) detection. Expectation, mean, variance, characteristic function, covariance and correlation, Markov and Chebychev inequalities, Jensen's inequality, conditional expectation. Minimum mean square error (MMSE) estimation, linear estimation, jointly Gaussian random variables.Extension of cdf, pdf, and pmf to more than two random variables, independence and conditional independence, covariance matrix, Gaussian random vectors, linear estimation - vector case. Modes of convergence, laws of large numbers, central limit theorem.

RECOMMENDED BOOKS:

1. Probability and Random Processes, Venkatarama Krishnan
2. Probability and Random Processes Aug 2, 2001by Geoffrey R. Grimmett and David R. Stirzaker
3. Probability, Statistics, and Random Processes for Engineers (4th Edition) Aug 20, 2011 by Henry Stark and John Woods

MA-6132 TIME SCALE CALCULUS

Course Title: TIME SCALE CALCULUS

Course Code: MATH 6132

Credit Hours: 3

Prerequisites: Linear Algebra

Course Contents: Basic Definitions on Time Scales, Some Dynamic Equations, Nabla Dynamic equations, Reimann and Lebesgue integration, delta and Nabla integrals, Inequalities on time scales.

RECOMMENDED BOOKS:

1. Martin Bohner & Allan Peterson, "Advances in Dynamic Equations in Time Scales", Birkhausar Boston, 2003.

2. R. P. Agarwal, “Difference equations and Inequalities”,Marcel Dekker, INC.,NewYork, 1992.
3. Martin Bohner & Allan Peterson, “Dynamic Equations on Time Scales, An Introduction with Applications”, Birkhausar Boston, 2001.

MA-6133 MEASURE AND INTEGRATION

Course Title: MEASURE AND INTEGRATION

Course Code: MATH 6133

Credit Hours: 3

Prerequisites: Real Analysis, Calculus

Course Contents: Set Theory: Functions, intersections and complements, Algebra of sets, the axiom of choice and infinite direct Products, countable sets, relations and equivalences, partial orderings and the maximal principle, well ordering and the countable ordinals.

The Real Number System: axioms for the real numbers, the natural and rational numbers as subsets of R, the extended real numbers, sequences of real numbers, open and closed sets of real numbers, continuous functions, Borel sets. Lebesgue Measure: countably additive measure, outer measure, measurable sets and Lebesgue measure, non-measurable set, measurable function, Littlewood’s three principle. The Lebesgue Integral: The Riemann integral, Lebesgue integral of a bounded function over a set of finite Measure, integral of a non-negative function, general Lebesgue integral, onvergence in measure.

Differentiation and Integration: Differentiation of monotone functions, functions of bounded variation, differentiation of an integral, absolute continuity, convex functions.

The classical Banach Spaces: the Lp spaces, Minkowski and Holder inequalities, convergence and completeness, approximation in Lp, bounded linear functional on the Lp spaces. General Measure and Integration Theory: measure spaces, measurable functions, integration, general convergence theorems, signed measure, the Radon-Nikodym Theorem, outer measure and measurability extension theorem, the Lebesgue-Stieltjes integral, product measure, integral operators,

RECOMMENDED BOOKS:

1. Real Analysis- 3rd Edition, By H. L. Royden, published by Macmillan Publishing Company, New York - 1988.
2. R. P. Agarwal, “Difference equations and Inequalities”,Marcel Dekker, INC.,NewYork, 1992.
3. Martin Bohner & Allan Peterson, “Dynamic Equations on Time Scales, An Introduction with Applications”, Birkhausar Boston, 2001.

MA-6134 ALGEBRAIC TOPOLOGY-I

Course Title: ALGEBRAIC TOPOLOGY-I

Course Code: MATH 6134

Credit Hours: 3

Prerequisites: Algebra & Topology

Course Contents: General Topology : Topological Spaces, Homeomorphism, Wedges and Smash Products, Topological Groups and Orbit Spaces, Mapping Spaces and Compact-Open Topology, Manifolds and Configuration Spaces

Elementary Homotopy Theory : Homotopy, Paths and Homotopy, Homotopy Equivalences and Contractible Spaces, Retraction, Deformation and Homotopy Extension Property, H-spaces and Co-H-spaces, Exact Sequences.

The Fundamental Groups : The fundamental Group, The Fundamental Group of the Circle, Free Products of Groups, The Seifert-Van Kampen Theorem, Applications to Cell Complexes.

The Covering Spaces : Covering Spaces, Lifting Theorem for Covering Spaces, Classification of Covering Spaces, Universal Covering.

RECOMMENDED BOOKS:

1. Algebraic Topology, Allen Hatcher, Cambridge University Press (2002)
2. A Basic Course In Algebraic Topology, W. Massey, Springer-Verlag (1993)
3. Topology: A first course, JR Munkres, Prentice-Hall (1975)

MA-6135 ALGEBRAIC TOPOLOGY-II

Course Title: ALGEBRAIC TOPOLOGY-II

Course Code: MATH 6135

Credit Hours: 3

Prerequisites: Algebra & Topology

Course Contents: Homology Theory: Delta-Complexes, Chain Complexes and Homology Groups,. Simplicial Homology. Singular Homology, Homotopy Invariance, The Equivalence of Simplicial and Singular Homology, Homology of a Point, $H_0(X)$ for Path-wise Connected Spaces, $H_1(X)$ versus $\pi_1(X)$, Homology of Convex Subspaces of R^n , Chain Maps and Chain Homotopy, Homotopy Invariance, Homology of Contractible Spaces, Exact Sequences and Excision, Mayer-Vietoris Sequence, Homology of Spheres, No-Retraction and Brouwer Fixed Point Theorems, Relative Homology, Brouwer Degree for Maps $S^n \to S^n$, Degree of the Antipodal Map

Cohomology Theory : Cohomology Groups: The Universal Coefficient Theorem, Cohomology of Spaces, Cup Product: The Cohomology Ring. A Kunneth Formula. Spaces with Polynomial Cohomology, Poincare Duality: Orientations and Homology, The Duality Theorem, Cup Product and Duality, Additional Topics: The Universal Coefficient Theorem for Homology, The General Kunneth Formula, H-Spaces and Hopf Algebras, The Cohomology of SO(n), Bockstein Homomorphisms, More About Ext. Transfer Homomorphisms, Local Coefficients

Recommended Books

1. Algebraic Topology, Allen Hatcher, *Cambridge University Press* (2002)
2. A Basic Course In Algebraic Topology, W. Massey, *Springer-Verlag* (1993)

MA-6136 GALOIS THEORY-I

Course Title: GALOIS THEORY-I

Course Code: MATH 6136

Credit Hours: 3

Prerequisites: Algebra & group Theory

Course Contents: Automorphisms of fields, Fixed subfields, The theorems of Dedekind on linear independence of automorphisms of fields, Galois extensions, Fundamental theorem of Galois Theory, Galois group of polynomials, Finite fields, Galois closure, Primitive element theorem, Cyclotomic fields, Cyclic and radical extensions, Solving equations by radicals.

RECOMMENDED BOOKS:

1. E. Aritn, *Galois Theory,* Dover Publications, Inc.,1942
2. D. S. Dummit and R. M. Foote, *Abstract Algebra, 2nd ed.,* John Wiley & Sons, Inc., New-York, 2003.
3. I. N. Stewart, *Galois Theory*, Chapman and Hall (2000)

MA-6137 GALOIS THEORY-II

Course Title: GALOIS THEORY-II

Course Code: MATH 6137

Credit Hours: 3

Prerequisites: Algebra & group Theory

Course Contents: Infinite Galois extensions, Topological groups, Krull Topology, Closed Subgroups, Inverse limits, Galois group and profinite groups, Fundamental theorem of infinite Galois Theory, Compactness of Galois group.

RECOMMENDED BOOKS:

1. Michael D. Fried and Moshe Jarden, *Field Arithmetic, 3rd ed.*, Springer, Berlin, Heidelberg, 2008.
2. N. Jacobson, *Lectures in Abstract Algebra III. Theory of Fields and Galois Theory,* D. van Nostrand Company, Inc., Princeton, New York, 1964.

MA-6138 MATHEMATICAL INEQUALITIES

Course Title: MATHEMATICAL INEQUALITIES

Course Code: MATH 6138

Credit Hours: 3

Prerequisites: Calculus, Real Analysis

Course Contents: Inequalities Involving Convex Functions, Inequalities Related to Hardy's Inequality, Opial-Type Inequalities, Poincaré- and Sobolev-Type Inequalities, Levin- and Lyapunov-Type Inequalities.

RECOMMENDED BOOKS:

1. B. G. Pachpatte, Mathematical Inequalities, North Holland Mathematical library, Elsevier 2005.
2. Sever S. Dragomir, Charles E.M. Pearce, Selected topics on Hermite-Hadamard type Inequalities & Applications, RGMIA, Melbourne, Australia 1998.
3. D. S. Mitrinovic, Elementary Inequalities, P. Noordhoff LTD GRONINGEN, The Netherlands, 1964.
4. E. F. Beckenbach and R. Bellman, Inequalities, Springer-Verlag, Berlin, 2nd Ed., 1983.

MA-6139 CONVEX FUNCTIONS

Course Title: CONVEX FUNCTIONS

Course Code: MATH 6139

Credit Hours: 3

Prerequisites: Calculus, Real Analysis

Course Contents: Convex functions on interval, young's inequality and its consequences, jensen's Inequality, smoothness properties, Integrla form of Jensen's inequality, Hermite-Hadamard inequality, Algebraic version, gamma & beta functions, Multiplicative convexity, Am-GM inequality, (M,N)-convex functions, convex functions on normed linear spaces, convex functions in higher dimensions, continuity & Differentiability of convex functions.

RECOMMENDED BOOKS:

1. Constantin P. Niculescu, Lars-Erik Persson, CONVEX FUNCTIONS AND THEIR APPLICATIONS A contemporary approach, Springer 2004.
2. E. M. Alfsen, Compact convex sets and boundary integrals, Springer-Verlag, Berlin, 1971.
3. V. Barbu and Th. Precupanu, Convexity and Optimization in Banach Spaces, Ed. academiei, Bucharest, and D. Reidel Publ. Co.,Dordrecht, 1986.
4. J. M. Borwein and A. S. Lewis, Convex Analysis and Nonlinear Optimization. Theory and Examples., Springer-Verlag, Berlin, 2000.

MA-6140 TOPICS IN VARIATIONAL AND QUASIVARIATIONAL INEQUALITIES

Course Title: TOPICS IN VARIATIONAL AND QUASIVARIATIONAL INEQUALITIES

Course Code: MATH 6140

Credit Hours: 3

Prerequisites: Calculus, Real Analysis

Course Contents: Affinely independent sets, *n*-simplices, simplicial sub-divisions of *n*-simplices; Sperner's Lemma, KKM Theorem, Brouwer Fixed Point Theorem. Ky Fan's KKM Mapping Principle, KKM Mapping; lower and upper semi-continuous single valued functions; convex, concave, quasi-convex and quasi-concave functions; Ky Fan's Minimax Inequality, equivalent formulations of Ky Fan's Minimax Inequality; Generalizations of Ky Fan's Minimax Inequality; Escaping sequences and generalization of minimax inequalities as application of escaping sequences; Generalized convex (G-convex) spaces, Minimax inequalities in G-convex spaces. Variational inequalities, generalized variational inequalites; generalized variational inequalities in reflexive Banach spaces; Set-valued lower and upper semi-continuous mappings; set-valued monotone and semi-monotone mappings; strong topology or the topology on the continuous dual space (of a topological vector space E) of uniform convergence on bounded sets in E, joint continuity in product spaces; Inward set of point y with respect to subset X of E; Kneser's Minimax Theorem.

Quasi-variational and generalized quasi-variational inequalities; Hahn Banach Separation Theorem, support of a function and partition of unity; Fan-Glicksberg Fixed Point Theorem (1952), Kakutani Fixed Point Theorem. Complementarity and generalized complementarity problems; Fixed Point Theorems, Hausdorff metric, non-expansive and pseudo-contractive mappings. Generalized Games (or Abstract Economies), KF-majorant, maximal element, Borglin-Keiding Theorem on maximal element, open graph, one person game, equilibrium points of one person and qualitative games, constraint and preference correspondences, single economy and strategy or choice sets, para-compact sets, continuous selection.

RECOMMENDED BOOKS:

1. Topological Methods for Set-Valued Nonlinear Analysis, by Enayet U. Tarafdar and Mohammad Showkat Rahim Chowdhury, published by World Scientific, London, Singapore – 2008.
2. Variational and Quasi-Variational Inequalities, by Claudio Baiocchi and Antonio Capelo, published by John Wiley and Sons, New York, Singapore. – 1984.

MA-6141 ADVANCE FUZZY ALGEBRA BCK Algebra

Course Title: ADVANCE FUZZY ALGEBRA BCK Algebra

Course Code: MATH 6141

Credit Hours: 3

Prerequisites: Algebra, Group Theory

Course Contents: Definition of BCK algebra. Examples. General properties of BCK algebra. Commutative BCK algebra. Ideal theory of BCK algebras. Definition, types and examples of ideals in BCK algebra. Self maps of BCK algebra. Types of ideals. Definition of self maps. Right and left self maps. Left regular maps and their general properties. Kernels and annihilators in BCK Algebras. Definition of kernel, annihilator and related theorems.

BCI Algebra

Classification of BCI Algebras:

Implicative, Positive implicative BCI algebras.

Classification and characterization of ideals in BCI Algebras:

Ideals in BCI algebra, Strong and weak ideals. Characterization of BCI-algebras of order 6 with BCK part of order 1, 2, 3, 4 and 5. Fuzzy p-ideals, fuzzy h-ideals, fuzzy α-ideals, fuzzy BCI-(implicative, positive implicative, commutative) ideals and their interrelationship and their characterization based on extension principle and level sets. Intuitionistic fuzzy sets and their applications in ideal theory of BCK/BCI-algebras.

Soft set theory

Soft sets, definition and examples from daily life. "Union", "Extended intersection", "Restricted Union", "Restricted intersection" "AND" and "OR" operations defined on soft sets illustrated by various examples. Soft algebras, soft ideals, soft p-ideals, soft h-ideals, soft α-ideals and soft BCI-(implicative, positive implicative, commutative ideals). Applications of fuzzy sets to these soft ideals and detail discussion of their properties. Intuitionistic fuzzy soft sets, intuitionistic fuzzy soft ideals and their properties. Decision making based on fuzzy soft sets and intuitionistic fuzzy soft sets.

Fuzzy hyperstructures

Hyperstructures, definition and examples. Set valued functions. Applications of hyperstructures in the ideal theory of BCK/BCI-algebras based on fuzzy sets. Fuzzy hyper BCK-algebras, fuzzy hyper BCI-algeras and their properties. Products and extensions of fuzzy hyperstructures.

RECOMMENDED BOOKS:

1. Geoge G. Klir and Bo Yuan, Fuzzy sets and Fuzzy Logic (Theory and Applications) (2010).
2. Meng, J. and Jun, Y.B.: BCK Algebras (Kyung Moon Sa. Co., Seoul., 1994).
3. Iseki, K. and Tanaka, S.: An Introduction to the Theory of BCK Algebra, Math. Japonica, 23, No.1(1978), 1-26.
4. Piergiulio Corsini and Violeta Leoreanu: Applications of Hyperstructure theory (2003).

MA-6142 APPROXIMATION THEORY

Course Title: APPROXIMATION THEORY

Course Code: MATH 6142

Credit Hours: 3

Prerequisites: Linear Algebra, Numerical Analysis

Course Contents: The approximation problem and existence of best approximations. The uniqueness of best approximations. Approximation operators. Polynomial interpolation. Uniform convergence of polynomial approximations. Least squares approximations. Properties of orthogonal polynomials, Order of convergence of polynomial approximations. Interpolation by piecewise polynomials. Chebyshev polynomials.

RECOMMENDED BOOKS:

1. Powell, M.J.D., (1988), "Approximation Theory and Methods", Cambridge university Press.
2. Cheney, E.W., (2002), "Introduction to approximation Theory", AMS Publications.
3. Achieser, N.I. and Akhiezer, N.I., (1992) "Theory of Approximation", Dover publications.
4. Jackson, D., (2002), (2^{nd} Ed.), "Introduction to Approximation Theory", AMS Publications.
5. Phillips, G.M., (2003), "Interpolation and Approximation by Polynomials", springer-Verlag.

MA-6143 CRYPTOGRAPHY

Course Title: CRYPTOGRAPHY

Course Code: MATH 6143

Credit Hours: 3

Prerequisites: group Theory, Algebra

Course Contents: Core Contents: Classic Ciphers and their analysis, Shanon's Information theory, Public Key Cryptography (PKC), Discrete Logarithm Problem (DLP), RSA Algorithm, Codes and cryptosystems

Detailed Contents:

a. **Classical Ciphers and their analysis:** Suit-case problem, Introduction to Cryptography and its applications, Advanced Topics in Number Theory (Solution of system of congruencies, Modular Arithmetic), classical ciphers and their deciphering
b. **Shanon's Information theory:** Shanon's theorem, Entropy, Redundancy and Unicity Distance, Mutual Information and Unconditionally Secure Systems
c. **Public Key Cryptography(PKC):** The Theoretical Model, Motivation and Set-up, Confidentiality, Digital Signature, Confidentiality and Digital Signature
d. **Discrete Logarithm Based Systems:**The Discrete Logarithm System, The Discrete Logarithm Problem(DLP), ElGamal's Public-Key Cryptosystems, ElGamal's Signature Scheme, How to Take Discrete Logarithms, Digital signature verification schemes
e. **RSA:**The RSA System, Setting Up the System, RSA for Privacy, RSA for Signatures

f. **Coding Theory Based Systems:** Introduction to coding theory, Repetition code and examples, decoding, Error-detection codes and Error-correcting codes, Setting Up the System, Encryption and Decryption

RECOMMENDED BOOKS:

1. Henk C.A. van Tilborg, Fundamentals of Cryptology, Springer; 2000.

2. J. Katz, Y. Lindell, Introduction to Modern Cryptography, Chapman and Hall/CRC, 2007.

3. A. J. Menezes, P. C. van Oorschot, S. A. Vanstone, Handbook of Applied Cryptography, CRC Press; 1st edition, 1996.

4. S. Ling, C. Xing, Coding theory: A First Course, Cambridge University Press, 2004.

MA-6144 MATHEMATICAL STATISTICS

Course Title: MATHEMATICAL STATISTICS

Course Code: MATH 6144

Credit Hours: 3

Prerequisites: Statistics

Course Contents: Calculus of distributions, moments, moment generating functions; multivariate distributions, marginal and conditional distributions, conditional expectation and variance operators, change of variable, multivariate normal distribution, exact distributions arising in statistics; weak convergence, convergence in distribution, weak law of large numbers, central limit theorem; statistical inference, likelihood, score and information; estimation, minimum variance unbiased estimation, the Cramer-Rao lower bound, exponential families, sufficient statistics, the Rao-Blackwell theorem, efficiency, consistency, maximum likelihood estimators, large sample properties; tests of hypotheses, most powerful tests, the Neyman-Pearson lemma, likelihood ratio, score and Wald tests, large sample properties.

RECOMMENDED BOOKS:

1. Mathematical Statistics John E. Freund and Ronald E. Walpole Prentice Hall, Englewood Cliffs, NJ.

2. Mathematical Statistics: Exercises and Solutions **2005th Edition by** Jun Shao (Author)

MA-6145 STACHASTIC PROCESS

Course Title: STACHASTIC PROCESS

Course Code: MATH 6145

Credit Hours: 3

Prerequisites: Statistics, Probability, Linear Algebra

Course Contents: Basic Probability:

Probability space and axioms, basic laws, conditional probability and Bayes rule, independence.

Random Variables:

Random variables, probability mass function (pmf), cumulative distribution function (cdf), probability density function (pdf), discrete, continuous, and mixed random variables, functions of random variables, generation of random variables.

Pairs of random variables, joint, marginal, and conditional distributions, maximum likelihood (ML) and maximum a posteriori probability (MAP) detection. Expectation, mean, variance, characteristic function, covariance and correlation, Markov and Chebychev inequalities, Jensen's inequality, conditional expectation. Minimum mean square error (MMSE) estimation, linear estimation, jointly Gaussian random variables.

Random Vectors:

Extension of cdf, pdf, and pmf to more than two random variables, independence and conditional independence, covariance matrix, Gaussian random vectors, linear estimation - vector case. Modes of convergence, laws of large numbers, central limit theorem.

Stochastic Processes:

Discrete-time and continuous-time random processes, memoryless, independent increment, Stochastic, Markov, and Gaussian random processes, point processes.

Stationary processes, autocorrelation functions and power spectral density (psd), white noise, bandlimited processes.

RECOMMENDED BOOKS:

1. Introduction to Stochastic Processes (Dover Books on Mathematics) January 24, 2013, Erhan Cinlar (Author)
2. A First Course in Stochastic Processes, 2nd Edition by Samuel Karlin (Author), Howard M. Taylor (Author)

MA-6146 RIEMANNIAN GEOMETRY

Course Title: RIEMANNIAN GEOMETRY

Course Code: MATH 6146

Credit Hours: 3

Prerequisites: Differential Geometry, Topology

Course Contents: Definition and examples of manifolds. Differential maps. Submanifolds. Tangents. Coordinate vector fields. Tangent spaces. Dual spaces. Multilinear functions. Algebra of tensors. Vector fields. Tensor fields. Integral curves. Flows. Lie derivatives. Brackets. Differential forms. Introduction to integration theo ry on manifolds. Riemannian and semi Riemannian metrics. Flat spaces. Affine connection. Parallel translations. Covariant differentiation of tensor fields. Curvature and Torsion tensors. Connection of a semi-Riemannian tensor. Killing equation and Killing ve ctor fields. Geodesics. Conformal transformations and the Weyl tensor.

RECOMMENDED BOOKS:

1. Bishop, R.L. a nd Goldberg, S.I.: Tensor Analysis on Manifolds (Dover publication, Inc. N.Y. 1980).
2. do Carmo, M.P.: Riemannian Geometry (Birkhauser, Boston, 1992).
3. Lovelock, D. and Rund, E. F.: Differential Forms and Variational Principles (John-Wiley, 1975).
4. Langwitz, D.: Differential and Riemannian Geometry (Academic Press, 1970).
5. Abraham, R., Marsden, J.E. and Ratiu, T.: Manifolds, Tensor Analysis and Applications (Addison Wesley, 1983).

MA-6147 GENERAL RELATIVITY-I

Course Title: GENERAL RELATIVITY-I

Course Code: MATH 6147

Credit Hours: 3

Prerequisites: linear Algebra

Course Contents: Original formulation of Special Relativity. Velocity addition in 3-d formulation. 4-Vector formalism. Poincare group. The null cone. Review of Electromagnetism. 4-Vector formulation of Maxwell’s equations. Special Relativity with small accelerations. The principles of General Rela tivity. The Einstein field equations. The stress-energy momentum tensor. The vacuum Einstein equations and the Schwarzschild solution. Birkhoff’s theorem. The Reissner-Nordstrom solution and the generalized Birkhoff’s theorem. The Kerr and the Kerr-Newmann solution. The Newtonian limit of Relativity. The Schwarzschild exterior solution and relati vistic equations of motion. The classical tests of Relativity and their current st atus. The Schwarzschild interior solution. Linearized gravity and gravitational waves. Foliations.

RECOMMENDED BOOKS:

1. Qadir, Asghar: Relativity: An Introduction to the Special Theory (World Scientific, 1989).
2. Stephani, Hans: General Relativity: An Introduction to the Theory of Gravitational Field (Cambridge University Press, 1990).
3. Wald, R.M.: General Relativity (The University of Chicago Press, 1984).
4. Misner, C.W., Thorne, K.S. and Wheeler, J.A.: Gravitation (W. H. Freeman and Co., 1973).
5. Plebanski, J. and Krasinski, A.: An Introduction to General Relativity and Cosmology (Cambridge University Press, 2006).

MA-6148 GENERAL RELATIVITY-II

Course Title: GENERAL RELATIVITY-II

Course Code: MATH 6148

Credit Hours: 3

Prerequisites: linear Algebra

Course Contents: Black holes. Coordinate and essential singularities. Horizons. Coordinates passing through horizons. The Kruskal and the Carter-Penrose (CP) diagrams for the Schwarzschild geometry. The maximal ex tension. The Einstein-Rosen bridge. Wormholes. The CP diagram for the RN metric. The no-hair and cosmic censorship hypotheses. Gravitational forces about black holes. Black hole thermodynamics. Observational status and central black holes. Kaluza-Klein theory. Problems of quantum gravity. Quantization in curved space backgrounds and Hawking radiation. Isometries. Homotheties and their significance in Relativity.

RECOMMENDED BOOKS:

1. Wald, R.M.: General Relativity (The University of Chicago Press, 1984).
2. Stephani, Hans: General Relativity: An Introduction to the Theory of Gravitational Field (Cambridge University Press, 1990).
3. Misner, C.W., Thorne, K.S. and Wheeler, J.A.: Gravitation (W. H. Freeman and Co., 1973).
4. Plebanski, J. and Kransinski, A.: An Introduction to General Relativity and Cosmology (Cambridge University Press, 2006).
5. Carroll, S.M.: An Introduction to General Relativity: Spacetime and Geometry (Addison Wesley, 2004).

MA-6149 COSMOLOGY

Course Title: COSMOLOGY

Course Code: MATH 6149

Credit Hours: 3

Prerequisites: linear Algebra

Course Contents: Historical background: Astronomy, Astrophysics and Cosmology. Observational facts about the Universe and its contents. The strong and weak forms of the cosmological principle. The Einstein and de Sitter mode ls of the Universe. The Hubble law and the Friedmann models. Steady state models. The hot big bang model. The microwave background. Discussion of the significance of the start of time. Fundamentals of High Energy Physics. The chronology and compositi on of the Universe. Non-baryonic dark matter. Problems of the standard model of cosmology. Structures a nd structure formation in the Universe. The cosmological constant. The inflationary solutions to the problems of the standard model. Later developments on the inflationary models. The Bianchi classification of homogeneous spacetimes. The Kasner (mixmaster) models of the Universe and the generic approach to the initial singularity. BKL-oscillations.

RECOMMENDED BOOKS:

1. Plebanski, J. and Kransinski, A.: An Introduction to General Relativity and Cosmology (Cambridge University Press, 2006).
2. Peebles, P.J.E.: Physical Cosmology (Princeton University Press, 1975).
3. Ryan, R. and Shepley, C.: Homogeneous Cosmology (Addison Wesley, 1982).
4. Abbott, L.F. and Pi, S.Y.: Inflationary Cosmology (World Scientific, 1986).
5. Joshi, P.S.: Global Aspects in Gravitation and Cosmology (Oxford University Press, 1993).

MA-6150 TOPOLOGICAL VECTOR SPACES

Course Title: COSMOLOGY

Course Code: MATH 6150

Credit Hours: 3

Prerequisites: Algebra, Topology, Vector Spaces

Course Contents: Vector space topologies, Subspaces. Quotient Spaces, Bounded Sets. Metriazability. Finite-dimentional spaces. Convex sets and seminorms. Locally convex topologically vector spaces. Barralled spaces and Bornological spaces. Duality. Dual systems and weak topologies. Compact Convex sets. Banach Alaoglu Theorem. The Krein-Millman Theorem, Compactness and finite dimension. Continuous linear mappings and spaces of linear mappings. Banach Steinhaus theorem. Approximation problem and compact maps.

RECOMMENDED BOOKS:

1. H.H. Schaefer, “Topological Vector Spaces” Sringer-Verlag, New York.
2. A. Robertson and W. Robertson “topological Vector Spaces” Cambridge.
3. G. Kothe “Topological Linear Spaces” Springer Verlag, New York.
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