**COURSE CURRICULUM**

**2023-ONWARD**

**FOR**

 **MS Mathematics**



**DEPARTMENT OF MATHEMATICAL SCIENCES**

**FACULTY OF BASIC SCIENCES & HUMANITIES**

**UNIVERSITY OF ENGINEERING & TECHNOLOGY, TAXILA**

**January 2024**

**MA-5101** **THEORY OF GROUP ACTIONS**

**Course Title: THEORY OF GROUP ACTIONS**

**Course Code: MA- 5101**

**Credit Hours: 3**

**Prerequisites: Algebra I**

**Course Contents:** Preliminaries, The theory of group actions: Permutation representations, Coset spaces, G-Space, Transitive G-Space, Orbits, Stabilizer, K-transitivity, Primitivity, Projective Spaces, Multiplicative group of finite fields, Extensions of finite fields, The Modular group, projective line over finite fields, Action of the Modular group, Projective and linear groups through actions, Parametrization of actions Modular group on projective line over finite fields.

**RECOMMENDED BOOKS:**

1. Coxeter, H.S.M. and Moser, W.O., Generators and Relations for Discrete Groups, Springer-Verlag, 1972.

2. Rose, S., A Course in Group Theory, Cambridge University Press, 1978. Dover Published 1994 (Reissued 2012).

3. Magnus, W., Karrass, A and Solitar, D., Combinatorial Group Theory, Dover Publications, 1976.

4. Johnson, D.L., Presentation of Groups, Cambridge Lecture Notes, 1976.

5. **John Meier**, **Groups, Graphs and Trees: An Introduction to the Geometry of Infinite Groups,** Cambridge University Press, 2008.

**MA-5102** **APPLIED LINEAR ALGEBRA-I**

**Course Title: APPLIED LINEAR ALGEBRA-I**

**Course Code: MA- 5102**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, Algebra I**

**Course Contents:** Existence of a linear transformation; Rank and nullity of a linear transformation; The algebra of linear transformations; Representation of transformations by matrices; Isomorphism; Dual Space of a vector space; Algebraic reflexivity; Hyperspace; The annihilator of a set; The transpose of a linear transformation. Concept of diagonalization of a linear transformation. Commutative linear algebra with identity over a field; Polynomial; Algebra of polynomials; Polynomial ideals; Prime factorization of a polynomial; Characteristic and minimal polynomials for a linear operator; Cayley-Hamilton theorem; Multiplicity of a root of a polynomial; Invariant subspaces; Simultaneous Triangulation and simultaneous diagonalization; Direct–sum decompositions; Invariant direct sums; The primary decomposition theorem.

**RECOMMENDED BOOKS:**

1. Apostol, T.M., “Linear Algebra”, Wiley International, 1997.
2. Hoffman, K.M. and Kunze, R. “Linear Algebra”, Prentice-Hall, 1971.
3. Karamat H. Dar, “Linear Algebra”, Carvan Book House-Lahore, 2007.
4. Lipschutz, S., “Schaum’s Outline of Linear Algebra”, McGraw Hill, 2002.
5. Mey, C.D., “Matrix Algebra and Applied Linear Algebra”, SIAM Publications, 2001.
6. Strang, G., “Introduction to Linear Algebra”, Wellesley-Cambridge Press, 1998.

**MA-5103** **VISCOUS FLUID FLOW**

**Course Title: VISCOUS FLUID FLOW**

**Course Code: MA-5103**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, Calculus, Discrete Mathematics**

**Course Contents:** Stress-strain rate relationships for a Newtonian Fluid, Navier-Stokes’ equations, Exact solutions of Navier-Stokes’ equations, Dimensional analysis, Dynamical similarity, Concepts of thermodynamics, Energy equation, Exact solutions for the problem of temperature distribution in a viscous flow, boundary layer concept and its governing equations. Exact solutions of boundary layer equations, Von Karman momentum integral equations and its application, Energy integral equation, Approximate solutions of steady boundary layer equations: Von Karman-pohlhausen method, Walz method, Thwaite’s method, Introduction to turbulent boundary layer flow.

**RECOMMENDED BOOKS:**

1. Schlichting, H., “Boundary Layer Theory (Seventh edition)” McGraw-Hill, 1979.
2. White, F. M., “Viscous Fluid Flow (Second edition)” McGraw-Hill, 1991.
3. Churchill, S.W., “Viscous Flows” Butterworth Publishers, 1988.
4. Hughes, W.F., “An Introduction to Viscous Flow” Hemisphere, N.Y.,1979.
5. Yaun, S.W., “Foundations of Fluid Mechanics” Prentice Hall, 1967.

**MA-5104** **PARTIAL DIFFERENTIAL EQUATIONS**

**Course Title: PARTIAL DIFFERENTIAL EQUATIONS**

**Course Code: MA-5104**

**Credit Hours: 3**

**Prerequisites: ODE’s,**

**Course Contents:** Cauchy’s Problems for Linear Second Order Equations in n-Independent Variables. Cauchy Kowalski Theorem. Characteristic surfaces. Adjoint operations, Bi characteristics. Spherical and Cylindrical Waves. Heat equation, Wave equation, Laplace equation, Maximum-Minimum Principle, Integral Transforms.

**RECOMMENDED BOOKS:**

1. Dennemyer, R., Introduction to Partial Differential Equations and Boundary Value Problems, McGraw-Hill Book Company, 1968.
2. Chester, C.R., Techniques in Partial Differential Equations, McGraw-Hill Book Company, 1971.

**MA-5105 FUZZY ALGEBRA**

**Course Title: FUZZY ALGEBRA**

**Course Code: MA-5105**

**Credit Hours: 3**

**Prerequisites: Algebra**

**Course Contents:** Introduction, The Concept of Fuzziness: Examples; Mathematical Modeling; Operations of fuzzy sets; Fuzziness as uncertainty. Algebra of Fuzzy Sets: Boolean Algebra and lattices; Equivalence relations and partitions; Composing mappings; Alpha-cuts; Images of alpha-level sets; Operations on fuzzy sets. Fuzzy Relations: Definition and examples; Binary Fuzzy relations Operations on Fuzzy relations; fuzzy partitions. Fuzzy Semigroups, Fuzzy ideals of semigroups; Fuzzy quasi-ideals; Fuzzy bi-ideals of Semigroups; Characterization of different classes of semigroups by the properties of their fuzzy ideals’ fuzzy quasi-ideals and fuzzy bi-ideals. Fuzzy Rings: Fuzzy ideals of rings; Prime; semiprime fuzzy ideals; Characterization of rings using the properties of fuzzy ideals.

**RECOMMENDED BOOKS:**

1. Hung T. Nguyen and Elbert A. Walker, A First Course in Fuzzy Logic, Chapman, and Hall/CRC 1999.
2. M. Ganesh, Introduction to Fuzzy Sets and Fuzzy Logic, Prentice-Hall of India, 2006.
3. John N. Mordeson and D. S. Malik, Fuzzy Commutative algebra, World Scientific,

 1998.

1. John N. Mordeson, D. S. Malik and Fuzzy Semigroups, Springer-Verlage, Nobuki Kuroki, 2003.

**MA–5106** **INTEGRAL TRANSFORMS**

**Course Title: INTEGRAL TRANSFORMS**

**Course Code: MA-5106**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, Calculus**

**Course Contents:** Introduction, Fourier Transforms, The Mellin Transforms, The Henkel Transforms, The Kanotorovich – Lebeder Transforms, The MehlerForch transform, Finite transforms, Generalised Functions.

**RECOMMENDED BOOKS**

1. Sneddon, I.N., “The use of Integral Transforms”, McGraw-Hill, 1972.
2. Davies, B., “Integral Transforms and their Applications”, Springer-Verlag, 2002.
3. Gelfand, I.M. and Vilenkin, N. Ya., “Generalized Functions Vol. I, II” Academic Press, 1977.
4. Debnath, L., “Integral Transforms and their Applications” CRC Press, 1995.
5. Titchmarsh, I., “The Theory of Functions”, Oxford university Press, 1970.

**MA-5107** **MATHEMATICAL INEQUALITIES**

**Course Title: MATHEMATICAL INEQUALITIES**

**Course Code: MA-5107**

**Credit Hours: 3**

**Prerequisites: Calculus, Real Analysis**

**Course Contents:** Inequalities Involving Convex Functions, Inequalities Related to Hardy’s Inequality, Opial-Type Inequalities, Poincaré- and Sobolev-Type Inequalities, Levin- and Lyapunov-Type Inequalities.

**RECOMMENDED BOOKS:**

1. B. G. Pachpatte, Mathematical Inequalities, North Holland Mathematical library,

 Elsevier 2005.

1. Sever S. Dragomir, Charles E.M. Pearce, Selected topics on Hermite-Hadamard type Inequalities & Applications, RGMIA, Melbourne, Australia 1998.
2. D. S. Mitrinovic, Elementary Inequalities, P. Noordhoff LTD GRONINGEN, The Netherlands, 1964.
3. E. F. Beckenbach and R. Bellman, Inequalities, Springer-Verlag, Berlin, 2nd Ed., 1983.

**MA-5108** **MATHEMATICAL STATISTICS**

**Course Title: MATHEMATICAL STATISTICS**

**Course Code: MA-5108**

**Credit Hours: 3**

**Prerequisites: Statistics**

**Course Contents:** Calculus of distributions, moments, moment generating functions; multivariate distributions, marginal and conditional distributions, conditional expectation and variance operators, change of variable, multivariate normal distribution, exact distributions arising in statistics; weak convergence, convergence in distribution, weak law of large numbers, central limit theorem; statistical inference, likelihood, score and information; estimation, minimum variance unbiased estimation, the Cramer-Rao lower bound, exponential families, sufficient statistics, the Rao-Blackwell theorem, efficiency, consistency, maximum likelihood estimators, large sample properties; tests of hypotheses, most powerful tests, the Neyman-Pearson lemma, likelihood ratio, score and Wald tests, large sample properties.

**RECOMMENDED BOOKS:**

1. Mathematical Statistics John E. Freund and Ronald E. Walpole Prentice Hall, Englewood Cliffs, NJ.

Mathematical Statistics: Exercises and Solutions **2005th Edition by** [Jun Shao](https://www.amazon.com/Jun-Shao/e/B001ITYDWI/ref%3Ddp_byline_cont_book_1) .

**MA-5109** **MATHEMATICAL TECHNIQUES FOR BOUNDARY VALUE PROBLEMS**

**Course Title: MATHEMATICAL TECHNIQUES FOR BOUNDARY VALUE**

 **PROBLEMS**

**Course Code: MA-5109**

**Credit Hours: 3**

**Prerequisites: ODE’s**

**Course Contents:** Green’s function method with applications to wave-propagation. **Perturbation method:** regular and singular perturbation techniques with applications, Variational methods, A survey of transform techniques, Wiener-Hopf technique with applications to diffraction problems.

**RECOMMENDED BOOKS:**

1. Nayfeh, A., Perturbation methods, 2008.
2. Stakgold, I., Boundary value problems of Mathematical Physics, 1968.
3. Noble, B., Methods based on the Wiener-Hopf technique for the solution of Partial Differential Equations, 1958.
4. Mitra, R., and Lee, S.W., Analytical Techniques in the Theory of Guided Waves,

 Macmillan Comp., New York 1971.

**MA-5110 COMPRESSIBLE FLUID FLOW**

**Course Title: COMPRESSIBLE FLUID FLOW**

**Course Code: MA-5110**

**Credit Hours: 3**

**Prerequisites: ODE’s, PDE’s**

**Course Contents:** Concepts of thermodynamics, Mach Number, Types of flows, Inviscid compressible flow, Continuity, Euler’s, Bernoulli’s, and Energy Equations, kelvin’s theorem, Potential function equation, Stream function, Shock waves, method of characteristics, Linearization and small perturbation methods, Viscous compressible flow, Navier Stokes’ equations for a compressible viscous flow, Exact solutions, Relation between velocity and temperature fields with and without pressure gradient, Energy equation laminar, boundary layer equation, Velocity and temperature relation in laminar boundary layer, Momentum and energy integral equations.

**RECOMMENDED BOOKS:**

1. Misos, R.V., “Mathematical Theory of Compressible Fluid Flow” Academic Press, 1966.
2. Chapman, A.J. and W.F. Walker, “Introductory Gas Dynamics” Holt N.Y, 1976.
3. Saad, M.A., “Compressible Fluid Flow”, Prentice-Hall, 1985.
4. Schreier, S., “Compressible Flow: John-Wiley & Sons, 1982.
5. Thompson, P.A., “Compressible Fluid Dynamics”, McGraw-Hill, 1972.

**MA-5111** **INTEGRAL EQUATIONS**

**Course Title: INTEGRAL EQUATIONS**

**Course Code: MA-5111**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, Calculus**

**Course Contents:** Existence Theorems, Integral Equations with L2 Kernels. Applications to partial differential equations. Integral transforms (Fourier Transforms, The Mellin Transforms, The Henkel Transforms, The Kantorovich – Lebeder Transforms, The Mehler Forch transform, Finite transforms), Wiener-Hopf Techniques, Generalized functions.

**RECOMMENDED BOOKS:**

1. Harry Hoch Stadl, Integral Equations, John Wiley, 1973.
2. Stakgold, I., Boundary Value Problems of Mathematical Physics, Macmillan, New York, 1968.

**MA-5112** **MAGNETOHYDRODYNAMICS-I**

**Course Title: MAGNETOHYDRODYNAMICS-I**

**Course Code: MA-5112**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, Calculus**

**Course Contents: Basic Equations:** Equations of electrodynamics, Equations of Fluid Dynamics, Ohm’s lawequations of magnetohydrodynamics.

**Motion of an Incompressible Fluid:** Motion of a viscous electrically conducting fluid withlinear current flow, steady state motion along a magnetic field, wave motion of an ideal fluid.

**Small Amplitude MHD Waves:** Magneto-sonic waves. Alfve’s waves, damping andexcitation of MHD waves, characteristics lines and surfaces.

**Simples Waves and Shock Waves in Magnetohydrodynamics:** Kinds of simple waves,distortion of the profile of a simple wave, discontinuities, simple and shock waves in relativistic magnetohydrodynamics, stability and structure of shock waves, discontinuities in various quantities, piston problem, oblique shock waves.

**RECOMMENDED BOOKS:**

1. Cowling, T.G., Magnetohydrodynamics, Interscience Publishers, 1963.
2. Kulikowsky, A.G., and Lyabimov, A.G., Magnetohydrodynamics, A.Wesley, 1965.
3. Alfve’s H., and Falthammar, C., Cosmical Electrodynamics, Clarendon Press, 1965.
4. Akhiezer et.al. Plasma Electrodynamics, Pergamon Press, 1975.
5. Kendale and Plumpton, C. Magnetohydrodynamics.
6. Anderson, J.E. Magnetohydrodynamics, Shock Waves, M.I.T. Press, 1975.

**MA-5113 NUMERICAL SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS**

**Course Title: NUMERICAL SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS**

**Course Code: MA-5113**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, Numerical Analysis**

**Course Contents:**

1. Boundary and Initial conditions, Polynomial approximations in higher dimensions.
2. **Finite Element Method:** The Galerkin method in one and more dimensions, Errorbound on the Galerkin method, The method of collocation, Error bounds on the collocation method, Comparison of efficiency of the finite difference and finite element method.
3. **Finite Difference Method:** Finite difference approximations.
4. Application to solution of linear and non-linear Partial Differential Equations appearing in Physical Problems.

**RECOMMENDED BOOKS:**

1. Strang G., and Fix G., An Analysis of Finite Element Method, Prentice Hall, New Jersey, 1973.

1. David S. Burnett., Finite Element Analysis from Concepts to Applications, Addision Wesley, 1987.
2. Myron B. Allen., Ismael Herrera and George F., Pinder Numerical Modeling in Science and Engineering, John Willy & Sons, New York, 1988.
3. Desai, G.S., Elementary Finite Element Method, Prentice Hall, Inc. 1988.

**MA-5114** **PERTURBATION METHODS IN FLUID MECHANICS**

**Course Title: PERTURBATION METHODS IN FLUID MECHANICS**

**Course Code: MA-5114**

**Credit Hours: 3**

**Prerequisites: Fluid Mechanics, Numerical Analysis**

**Course Contents: L**imit procedures, Order notation, Definition, Matching principle, simple algebraic and differential equations, thin aero foil theory, Supersonic aero foil theory, Inviscid shallow water waves, P-L-K; Vander Pol equation, Non-linear oscillations, boundary layers flow, Low Reynolds number flow, Lubrication theory, Steepest descents and second order ordinary differential equations.

**RECOMMENDED BOOKS:**

1. Dyke, M.V. “Perturbation Methods in fluid Mechanics”, Parabolic Press, 1975.
2. Cole, J.D. and Kevorkian, J., “Perturbation Methods in Applied Mathematics”, Springer Verlag, 1985.
3. Nayfeh, A.H., “Perturbation Methods”, John Wiley and sons, 2000.
4. Wasow, W.R., “Asymptotic Expansions for O.D.E.”, R.E. Dreiger Publishing Company, 1985.
5. Copson, E.T., “Asymptotic Expansions”, SIAM Publications, 1980.

**MA-5115 THEORY OF SPLINES-I**

**Course Title: THEORY OF SPLINES-I**

**Course Code: MA-5115**

**Credit Hours: 3**

**Prerequisites: ODE’s, Linear Algebra**

**Course Contents:** Spline function of one variable: Interpolating cubic splines, smoothing cubic splines. Spline functions of two variables: Interpolating B-cubic splines, smoothing bicubic splines. Geometric splines: Spline curves, Bezier curves, B-spline curves. Spline surfaces: Bezier surfaces, B-spline surfaces.

**RECOMMENDED BOOKS:**

1. Eugene V. Shikin & Alexander-I Plis, “Handbook on splines”, CRC Press, 1995.
2. Larry L. Shumaker, “Spline Functional: Basic Theory”, 3rd ed, Cambridge University Press, 2007.
3. Carl de Boor, “A Practical Guide to Splines” Springer, 2000.
4. J.H. Ahlberg& E.N Nilson, “The Theory of Splines & their applications” Academic Press, 1967.
5. Charles K. Chui, “Multivariable Splines”, SIAM Publications, 1991.

**MA-5116 ADVANCED OPERATIONS RESEARCH-I**

**Course Title: ADVANCED OPERATIONS RESEARCH-I**

**Course Code: MA-5116**

**Credit Hours: 3**

**Prerequisites: Linear Algebra**

**Course Contents:** Simplex algorithm and sensitivity analysis, Dual simplex method, Interior-Point Algorithm, Shortest-path problem, Integer programming, Graphical illustration of nonlinear programming problems One-dimensional unconstraint optimization, Multivariable unconstraint optimization, Steepest Descent method, Newton’s method, The Karush-Kuhn-Tucker conditions, Quadratic programming.

**RECOMMENDED BOOKS:**

1. Hillier, F.and Lieberman, G.J., “Introduction to Operations Research”, McGraw Hill, (7th Ed.), 2001.
2. Taha, H.A., “Operations Research: An Introduction”, McGraw Hill, (7th Ed.), 2002.
3. Bertsimas, D. and Tsitsikhis, J.N., “Introduction to Linear Optimization”, Athena Scientific Publications, 1997.
4. Sofer, A. and Nash, S.G., “Linear and Nonlinear programming”, McGraw Hill, 1995.
5. Radin, R.L., “Optimization in Operations Research”, Prentice Hall, 1997.

**MA-5117** **SEMIGROUP THEORY**

**Course Title: SEMIGROUP THEORY**

**Course Code: MA-5117**

**Credit Hours: 3**

**Prerequisites: Group Theory**

**Course Contents:** Introductory ideas: Basic definitions, Cyclic semigroups; Ordered sets, semi lattices and lattices. Binary relations; Equivalences; Congruences; Free semigroups; Green’s Equivalences; L, R, H, J and D; Regular semigroups, O-Simple semigroups; Simple and O-Simple semigroups; Rees’s theorem; Primativeidempotents; Completely O-Simple semigroups; Finite congruence-free semigroups, Union of groups; Bands; Free bands; varieties of bands, Inverse semigroups, Congruences on inverse semigroups; Fundamental inverse semigroups; Bi-simple and simple inverse semigroups. Orthodox semigroups; Basic properties; The structure of orthodox semigroups.

**RECOMMENDED BOOKS:**

1. A.H. Clifford and G.B. Preston, The Algebraic Theory of Semigroups; Vol. I& II.

 AMS Math. Surveys, 1961 and 1967.

1. J.M. Howie, An Introduction to Semigroup Theory, Academic Press 1976.

**MA-5118** **THEORY OF ORDINARY DIFFERNTIAL EQUATIONS**

**Course Title: THEORY OF ORDINARY DIFFERNTIAL EQUATIONS**

**Course Code: MA-5118**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, ODE’s**

**Course Contents:**

**Systems of Linear Differential Equations:**

Linear systems, Solution matrix, Fundamental solution matrix, Eigen values, Different methods for solving the homogeneous systems, Autonomous systems, Critical points and periodic solutions, Stability of linear systems, Linearization and local stability, Applications of autonomous systems.

**Partial Differential Equations:**

Classification of equations, Linear and quasi linear first order equations, Method of Lagrange, Cauchy problem for first order and higher order equations in n-independent variables, Normal forms, Hyperbolic, Parabolic, and Elliptic equations.

**RECOMMENDED BOOKS:**

1. Dennis G. Zill and Michael R. Cullen, “Differential Equations with Boundary Value Problems”, PWS Publishing Co., Boston, USA, (3rd Ed.), 1993.
2. P. A. Glending, “Stability, Instability and Chaos”, Cambridge University Press, 1994.
3. Richard E. William Son, “Introduction to Differential Equations and Dynamical Systems”, McGraw-Hill International Ed, 1997.
4. Rene Denne Meyer, “Introduction to Partial Differential Equations Boundary Value Problems”, McGraw-Hill Book Co, 1968.
5. George F. Carrier and Carl E. Pearson, “Partial Differential Equations”, Academic Press, New York, 1976.

**MA-5119** **GRAPH THEORY**

**Course Title: GRAPH THEORY**

 **Course Code: MA-5119**

**Credit Hours: 3**

**Prerequisites: Linear Algebra**

**Course Contents:** Fundamentals. Definition. Tree puzzles, Paths, cycles, and trees. Hamilton cycles and Euler circuits, Planer graphs, Flows, Connectivity and Matching Network flows. Bipartite graphs and their properties. Connectivity and Menger’s theorem. External problems paths and Complete Subgraphs. Hamilton path and cycles. Coloring. Vertex coloring Edge coloring. Brook`s theorem, Chromatic polynomials. Graph on surfaces, Dual graphs, infinite graphs. Digraphs. Eulerian digraphs and tournaments. Marker chains. Hall`s marriage theorem and its applications. Transversal theory. Introduction to Matroids, Matroids and graphs. Matroids and transversals

**RECOMMENDED BOOKS:**

1. Bollobas, B.: Graph Theory (Springer Verlag, New York, 1979).
2. Wilson, R.J.: Introduction to Graph Theory (Longman London, 1979).
3. Bollobas, B.: Modern Graph Theory (Springer Verlag, NY, 2002).
4. Biggs, N.: Algebraic Graph Theory (Cambridge University Press, 1974).
5. Gross, J.L and Yellen, J.: Graph Theory and its Applications (Chapman and Hall, 2005).

**MA-5120** **PROBABILITY AND RANDOM PROCESS**

**Course Title: PROBABILITY AND RANDOM PROCESS**

 **Course Code: MA-5120**

**Credit Hours: 3**

**Prerequisites: Probability & Statistics**

**Course Contents:** Probability space and axioms, basic laws, conditional probability and Bayes rule, independence. Random variables, Probability Mass Function (PMF), Cumulative Distribution Function (CDF), Probability Density Function (PDF), discrete, continuous, and mixed random variables, functions of random variables, generation of random variables. Pairs of random variables, joint, marginal, and conditional distributions, maximum likelihood (ML) and maximum a posteriori probability (MAP) detection. Expectation, mean, variance, characteristic function, covariance and correlation, Markov and Chebyshev inequalities, Jensen’s inequality, conditional expectation. Minimum mean square error (MMSE) estimation, linear estimation, jointly Gaussian random variables, Extension of CDF, PDF, and PMF to more than two random variables, independence and conditional independence, covariance matrix, Gaussian random vectors, linear estimation - vector case. Modes of convergence, laws of large numbers, central limit theorem.

**RECOMMENDED BOOKS:**

1. Probability and Random Processes, [Venkataraman Krishnan](http://eu.wiley.com/WileyCDA/Section/id-302479.html?query=Venkatarama+Krishnan).
2. [Probability and Random Processes](https://www.amazon.com/Probability-Random-Processes-Geoffrey-Grimmett/dp/0198572220/ref%3Dsr_1_1?s=books&ie=UTF8&qid=1475086416&sr=1-1&keywords=probability+and+random+processes) Aug 2, 2001by Geoffrey R. Grimmett and [David R. Stirzaker](https://www.amazon.com/David-R.-Stirzaker/e/B007T19ZKY/ref%3Dsr_ntt_srch_lnk_1?qid=1475086416&sr=1-1).
3. [Probability, Statistics, and Random Processes for Engineers (4th Edition)](https://www.amazon.com/Probability-Statistics-Random-Processes-Engineers/dp/0132311232/ref%3Dsr_1_5?s=books&ie=UTF8&qid=1475086416&sr=1-5&keywords=probability+and+random+processes) Aug 20, 2011, by Henry Stark and John Woods.

**MA-5121** **MEASURE AND INTEGRATION**

**Course Title: MEASURE AND INTEGRATION**

**Course Code: MA-5121**

**Credit Hours: 3**

**Prerequisites: Real Analysis, Calculus**

**Course Contents:** Set Theory: Functions, intersections and complements, Algebra of sets, the axiom of choice and infinite direct Products, countable sets, relations and equivalences, partial orderings, and the maximal principle, well ordering and the countable ordinals.

The Real Number System: axioms for the real numbers, the natural and rational numbers as subsets of R, the extended real numbers, sequences of real numbers, open and closed sets of real numbers, continuous functions, Borel sets. Lebesgue Measure: countably additive measure, outer measure, measurable sets and Lebesgue measure, non-measurable set, measurable function, Littlewood’s three principle. The Lebesgue Integral: The Riemann integral, Lebesgue integral of a bounded function over a set of finite Measure, integral of a non-negative function, general Lebesgue integral, Convergence in measure.

Differentiation and Integration: Differentiation of monotone functions, functions of bounded variation, differentiation of an integral, absolute continuity, convex functions.

The classical Banach Spaces: the spaces, Minkowski’s and Holder’s inequalities, Convergence and Completeness, Approximation in Lp, Bounded linear functional on the Lp spaces. General Measure and Integration Theory: measure spaces, measurable functions, integration, general convergence theorems, signed measure, the Radon-Nikodym Theorem, outer measure, and measurability extension theorem, the Lebesgue-Stieltjes integral, product measure, integral operators,

**RECOMMENDED BOOKS:**

1. Real Analysis- 3rd Edition, By H. L. Royden, published by Macmillan Publishing Company, New York - 1988.
2. R. P. Agarwal, “Difference equations and Inequalities”, Marcel Dekker, INC., New York, 1992.
3. Martin Bohner & Allan Peterson, “Dynamic Equations on Time Scales, An Introduction with Applications”, [Birkhäuser,](https://www.springer.com/gp/birkhaeuser) Boston, 2001.

**MA-5122** **ALGEBRAIC TOPOLOGY-I**

**Course Title: ALGEBRAIC TOPOLOGY-I**

**Course Code: MA-5122**

**Credit Hours: 3**

**Prerequisites: Algebra & Topology**

**Course Contents: General Topology:** Topological Spaces, Homeomorphism, Wedges and Smash Products, TopologicalGroups and Orbit Spaces, Mapping Spaces and Compact-Open Topology, Manifolds and Configuration Spaces

**Elementary Homotopy Theory:** Homotopy, Paths and Homotopy, Homotopy Equivalences andContractible Spaces, Retraction, Deformation and Homotopy Extension Property, H-spaces and Co-H-spaces, Exact Sequences.

**The Fundamental Groups:** The fundamental Group, The Fundamental Group of the Circle, FreeProducts of Groups, The Seifert-Van Kampen Theorem, Applications to Cell Complexes.

**The Covering Spaces:** Covering Spaces, Lifting Theorem for Covering Spaces, Classification ofCovering Spaces, Universal Covering.

**RECOMMENDED BOOKS:**

1. Algebraic Topology, Allen Hatcher, Cambridge University Press (2002)
2. A Basic Course in Algebraic Topology, W. Massey, Springer-Verlag (1993)
3. Topology: A first course, JR Munkres, Prentice-Hall (1975)

**MA-5123** **GALOIS THEORY-I**

**Course Title: GALOIS THEORY-I**

**Course Code: MA-5123**

**Credit Hours: 3**

**Prerequisites: Algebra & group Theory**

**Course Contents:** Automorphisms of fields, Fixed subfields, Theorems of Dedekind on linear independence of automorphisms of fields, Galois extensions, Fundamental theorem of Galois Theory, Galois group of polynomials, Finite fields, Galois closure, Primitive element theorem, Cyclotomic fields, Cyclic and radical extensions, Solving equations by radicals.

**RECOMMENDED BOOKS:**

1. E. Aritn, *Galois Theory,* Dover Publications, Inc.,1942.
2. D. S. Dummit and R. M. Foote*, Abstract Algebra, 2nd ed.,* John Wiley & Sons, Inc., New-York, 2003. I. N. Stewart, *Galois Theory*, Chapman, and Hall (2000).

**MA-5124 TOPICS IN VARIATIONAL AND QUASI VARIATIONAL INEQUALITIES**

**Course Title: TOPICS IN VARIATIONAL AND QUASI VARIATIONAL INEQUALITIES**

**Course Code: MA-5124**

**Credit Hours: 3**

**Prerequisites: Calculus, Real Analysis**

**Course Contents:** Affinely independent sets, *n*-simplices, simplicial sub-divisions of *n*-simplices; Spener’s Lemma, KKM Theorem, Brouwer Fixed Point Theorem. Ky Fan’s KKM Mapping Principle, KKM Mapping; lower and upper semi-continuous single valued functions; convex, concave, quasi-convex and quasi-concave functions; Ky Fan’s Minimax Inequality, equivalent formulations of Ky Fan’s Minimax Inequality; Generalizations of Ky Fan’s Minimax Inequality; Escaping sequences and generalization of minimax inequalities as application of escaping sequences; Generalized convex (G-convex) spaces, Minimax inequalities in G-convex spaces. Variational inequalities, generalized variational inequalities; generalized variational inequalities in reflexive Banach spaces; Set-valued lower and upper semi-continuous mappings; set-valued monotone and semi-monotone mappings; strong topology or the topology on the continuous dual space (of a topological vector space E) of uniform convergence on bounded sets in E, joint continuity in product spaces; Inward set of point y with respect to subset X of E; Kneser’s Minimax Theorem.

Quasi-variational and generalized quasi-variational inequalities; Hahn Banach Separation Theorem, support of a function and partition of unity; Fan-Glicksberg Fixed Point Theorem (1952), Kakutani Fixed Point Theorem. Complementarity and generalized complementarity problems; Fixed Point Theorems, Hausdorff metric, non-expansive and pseudo-contractive mappings. Generalized Games (or Abstract Economies), KF-majorant, maximal element, Borglin-Keiding Theorem on maximal element, open graph, one person game, equilibrium points of one person and qualitative games, constraint and preference correspondences, single economy and strategy or choice sets, para-compact sets, continuous selection.

**RECOMMENDED BOOKS:**

1. Topological Methods for Set-Valued Nonlinear Analysis, by Enayet U. Tarafdar and Mohammad Showkat Rahim Chowdhury, published by World Scientific, London, Singapore – 2008.
2. Variational and Quasi-Variational Inequalities, by Claudio Baiocchi and Antonio Capelo, published by John Wiley and Sons, New York, Singapore. – 1984.

**MA-5125** **CRYPTOGRAPHY**

**Course Title: CRYPTOGRAPHY**

**Course Code: MA-5125**

**Credit Hours: 3**

**Prerequisites: group Theory, Algebra**

**Course Contents: Core Contents:** Classic Ciphers and their analysis, Shannon’s Information theory,Public KeyCryptography (PKC), Discrete Logarithm Problem (DLP), RSA Algorithm, Codes and cryptosystems

**Detailed Contents:**

1. **Classical Ciphers and their analysis:** Suit-case problem, Introduction to Cryptography and itsapplications, Advanced Topics in Number Theory (Solution of system of congruencies, Modular Arithmetic), classical ciphers and their deciphering
2. **Shannon’s Information theory:** Shannon’s theorem, Entropy, Redundancy and Unicity Distance, Mutual Information and Unconditionally Secure Systems
3. **Public Key Cryptography (PKC):** The Theoretical Model, Motivation and Set-up,Confidentiality, Digital Signature, Confidentiality and Digital Signature
4. **Discrete Logarithm Based Systems:** The Discrete Logarithm System, The Discrete LogarithmProblem (DLP), ElGamal's Public-Key Cryptosystems, ElGamal's Signature Scheme, How to Take Discrete Logarithms, Digital signature verification schemes.
5. **RSA:** The RSA System, Setting Up the System, RSA for Privacy, RSA for Signatures
6. **Coding Theory Based Systems:** Introduction to coding theory,Repetition code and examples,decoding, Error-detection codes and Error-correcting codes, Setting Up the System, Encryption and Decryption.

**RECOMMENDED BOOKS:**

1. Henk C.A. van Tilborg, Fundamentals of Cryptology, Springer; 2000.
2. J. Katz, Y. Lindell, Introduction to Modern Cryptography, Chapman, and Hall/CRC, 2007.
3. A. J. Menezes, P. C. van Oorschot, S. A. Vanstone, Handbook of Applied Cryptography, CRC Press; 1st edition, 1996.
4. S. Ling, C. Xing, Coding theory: A First Course, Cambridge University Press, 2004.

**MA-5126** **GENERAL RELATIVITY-I**

**Course Title: GENERAL RELATIVITY-I**

**Course Code: MA-5126**

**Credit Hours: 3**

**Prerequisites: linear Algebra**

**Course Contents:** Original formulation of Special Relativity. Velocity addition in 3-d formulation. 4-Vector formalism. Poincare group. The null cone. Review of Electromagnetism. 4-Vector formulation of Maxwell’s equations. Special Relativity with small accelerations. The principles of General Relativity. The Einstein field equations. The stress-energy momentum tensor. The vacuum Einstein equations and the Schwarzschild solution. Birkhoff’s theorem. The Reissner-Nordstrom solution and the generalized Birkhoff’s theorem. The Kerr and the Kerr-Newmann solution. The Newtonian limit of Relativity. The Schwarzschild exterior solution and relativistic equations of motion. The classical tests of Relativity and their current status. The Schwarz-shield interior solution. Linearized gravity and gravitational waves. Foliations.

**RECOMMENDED BOOKS:**

1. Qadir, Asghar: Relativity: An Introduction to the Special Theory (World Scientific, 1989).
2. Stephani, Hans: General Relativity: An Introduction to the Theory of Gravitational Field (Cambridge University Press, 1990).
3. Wald, R.M.: General Relativity (The University of Chicago Press, 1984).
4. Misner, C.W., Thorne, K.S. and Wheeler, J.A.: Gravitation (W. H. Freeman and Co., 1973).
5. Plebanski, J. and Krasinski, A.: An Introduction to General Relativity and Cosmology (Cambridge University Press, 2006).

**MA-5127** **COSMOLOGY**

**Course Title: COSMOLOGY**

**Course Code: MA-5127**

**Credit Hours: 3**

**Prerequisites: linear Algebra**

**Course Contents:** Historical background: Astronomy, Astrophysics and Cosmology. Observational facts about the Universe and its contents. The strong and weak forms of the cosmological principle. The Einstein and de Sitter mode ls of the Universe. The Hubble law and the Friedmann models. Steady state models. The hot big bang model. The microwave background. Discussion of the significance of the start of time. Fundamentals of High Energy Physics. The chronology and composition of the Universe. Non-baryonic dark matter. Problems of the standard model of cosmology. Structures and structure formation in the Universe. The cosmological constant. The inflationary solutions to the problems of the standard model. Later developments on the inflationary models. The Bianchi classification of homogeneous spacetimes. The Kasner (Mixmaster) models of the Universe and the generic approach to the initial singularity. BKL-oscillations.

**RECOMMENDED BOOKS:**

1. Plebanski, J. and Krasinski, A.: An Introduction to General Relativity and Cosmology (Cambridge University Press, 2006).
2. Peebles, P.J.E.: Physical Cosmology (Princeton University Press, 1975).
3. Ryan, R. and Shepley, C.: Homogeneous Cosmology (Addison Wesley, 1982).
4. Abbott, L.F. and Pi, S.Y.: Inflationary Cosmology (World Scientific, 1986).
5. Joshi, P.S.: Global Aspects in Gravitation and Cosmology (Oxford University

 Press, 1993).

**MA-5128 TOPOLOGICAL VECTOR SPACES**

**Course Title: TOPOLOGICAL VECTOR SPACES**

**Course Code: MA-5128**

**Credit Hours: 3**

**Prerequisites: Algebra, Topology, Vector Spaces**

**Course Contents:** Vector space topologies, Subspaces. Quotient Spaces, Bounded Sets. Metriazability. Finite-dimensional spaces. Convex sets and semi norms. Locally convex topologically vector spaces. Barralled spaces and Bornological spaces. Duality. Dual systems and weak topologies. Compact Convex sets. Banach Alaoglu Theorem. The Krein-Millman Theorem, Compactness, and finite dimension. Continuous linear mappings and spaces of linear mappings. Banach Steinhaus theorem. Approximation problem and compact maps.

**RECOMMENDED BOOKS:**

1. H.H. Schaefer, “Topological Vector Spaces” Sringer-Verlag, New York.
2. A. Robertson and W. Robertson “topological Vector Spaces” Cambridge.
3. G. Kothe “Topological Linear Spaces” Springer Verlag, New York.
4. S.M. Khaleelulla, (1996) “Counter Examples in Topological Vector Spaces” Lecture Notes in Mathematics. Sringer-Verlag, New York.
5. A.L. Peressini “Ordered Topological Vector Spaces” Harper & Row, New York.
6. J. L. Kelley. Isaac Namioka “Linear Topological Spaces” Sringer Verlag, New York.