**COURSE CURRICULUM**

**2023-ONWARD**

**FOR**

**PhD Mathematics**



**DEPARTMENT OF MATHEMATICAL SCIENCES**

**FACULTY OF BASIC SCIENCES & HUMANITIES**

**UNIVERSITY OF ENGINEERING & TECHNOLOGY, TAXILA**

**January 2024**

**MA-6101** **THEORY OF GROUP GRAPHS**

**Course Title: THEORY OF GROUP GRAPHS**

**Course Code: MA-6101**

**Credit Hours: 3**

**Prerequisites: Algebra I**

Course Contents: **Generators and relations for certain groups, Finite presentations of group, Factor groups, Free groups, Tietze transformations, Dicyclic groups, Quaternion group, Extensions of cyclic groups, Systematic enumeration of cosets,** Graphs, Cayley diagram Schreier coset diagram, Coset diagram for the Modular group, graphs for group actions, projective special linear group and its action on real, rational and irrational fields, Graphical representations of Mobius, Orthogonal, Affine and Euclidean groups.

RECOMMENDED BOOKS:

1. Coxeter, H.S.M. and Moser, W.O., Generators and relations for discrete groups, Springer-Verlag, 1972.
2. Rose, S., A Course in Group Theory, Cambridge University Press, 1978. Dover Published 1994 (Reissued 2012).
3. Magnus, W., Karrass, A and Solitar, D., Combinatorial group theory, Dover Publications, 1972.
4. [Gary Chartrand](http://www.amazon.com/Gary-Chartrand/e/B0027MLJP4/ref=dp_byline_cont_book_1)  and [Ping Zhang](http://www.amazon.com/s/ref=dp_byline_sr_book_2?ie=UTF8&field-author=Ping+Zhang&search-alias=books&text=Ping+Zhang&sort=relevancerank), A First Course in Graph Theory,  Dover

Publications, 2012.

1. John Meier, Groups, Graphs and Trees: An Introduction to the Geometry of

Infinite Groups, Cambridge University Press, 2008.

**MA-6102** **ADVANECD MATHEMATICAL MODELLING**

**Course Title: ADVANECD MATHEMATICAL MODELLING**

**Course Code: MA-6102**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, Calculus, Discrete Mathematics**

**Course Contents:** Ordinary differential equations, Modelling with first order differential equations, Population growth model, Competitive Hunter model, Predator Prey model, Introduction to compartment model, Modelling with 2nd order differential equations, Application to biological system, Modelling with wave, heat, and Laplace equations. Partial differential equations methodology of mathematical modelling, Traffic flow, RC Circuits, Introduction to delay differential equations, modelling technique in case of delay differential equations (Cancer model, Baroreceptor model)

**RECOMMENDED BOOKS:**

1. Mathematical Modelling by Jagat Narain Kapur Edition 1998.
2. Cardiovascular and Respiratory Systems: Modeling, Analysis, and Control by Jerry J.

Batzel, Franz Kappel, Daniel Schneditz, Hien T. Tran Edition October 2005**.**

1. Cardiovascular Mathematics: Modeling and simulation of the circulatory system by von Luca Formaggia, Alfio Quarteroni, Alessandro Veneziani.
2. Applied Mathematical Models in Human Physiology [byJohnny T. Ottesen](http://www.amazon.com/Johnny-T.-Ottesen/e/B001I7Q04O/ref=ntt_athr_dp_pel_1) , [Mette S.](http://www.amazon.com/s/ref=ntt_athr_dp_sr_2?_encoding=UTF8&field-author=Mette%20S.%20Olufsen&search-alias=books&sort=relevancerank) [Olufsen](http://www.amazon.com/s/ref=ntt_athr_dp_sr_2?_encoding=UTF8&field-author=Mette%20S.%20Olufsen&search-alias=books&sort=relevancerank) and [Jesper K. Larsen.](http://www.amazon.com/s/ref=ntt_athr_dp_sr_3?_encoding=UTF8&field-author=Jesper%20K.%20Larsen&search-alias=books&sort=relevancerank)

**MA-6103 APPLIED FUNCTIONAL ANALYSIS-I**

**Course Title: APPLIED FUNCTIONAL ANALYSIS-I**

**Course Code: MA-6103**

**Credit Hours: 3**

**Prerequisites: Algebra**

**Course Contents:** Bounded linear operators, Bounded linear functionals, Riesz theorem for functionals on Hilbert spaces, Riesz representation theorem, Existence theorem for Hilbert-adjoint operator, Properties of Hilbert-adjoint operators, Sequences of self-adjoint operators, Unitary and normal operators. Hahn Banach theorems, Norm of the adjoint operator, Relation between the adjoint operator and the Hilbert-adjoint operator, Reflexive spaces, Baire’s category theorem; Uniform boundedness theorem; Open mapping theorem; Closed graph theorem; Banach fixed point theorem; Properties of Banach algebras.

**RECOMMENDED BOOKS:**

1. F. Riesz and Nagy, “Functional Analysis”, Frederick Unger publishing Co, 1965.
2. E. Kreyszig, “Introductory Functional Analysis with Applications”, John, Wiley and Sons, New York, 1989.
3. A.E. Taylor, “Introduction to Functional Analysis”, Wiley International Edition, New York, 1957.
4. W. Rudin, “Functional Analysis”, McGraw-Hill, Inc., New York, 1991.
5. M.T. Nair, “Functional Analysis”, Prentice Hall of India, New Delhi, 2002.

**MA-6104** **NON-NEWTONIAN FLUID MECHANICS**

**Course Title: NON-NEWTONIAN FLUID MECHANICS**

**Course Code: MA-6104**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, Calculus**

**Course Contents:** Classification of Non-Newtonian Fluids, Rheological formulae (Time-independent fluids, Thixotropic fluids, and viscoelastic fluids), Variable viscosity fluids, Cross viscosity fluids, The deformation rate, Viscoelastic equation, Materials with short memories, Time dependent viscosity. The Rivlin-Ericksen fluid, Basic equations of motion in rheological models. The linear viscoelastic liquid, Couette flow, Poiseuille flows. The current semi-infinite field, Axial oscillatory tube flow, Angular oscillatory motion, Periodic transients, Basic equations in boundary layer theory, Orders of magnitude, Truncated solutions for viscoelastic flow, Similarity solutions, Turbulent boundary layers, Stability analysis.

**RECOMMENDED BOOKS:**

1. John Harris, Theology and Non-Newtonian Flow, Longman Inc, New York 1977.
2. W.R. Schowalter, Mechanics of Non-Newtonian fluids, New York, Pergamon Press 1978.
3. R.B. Birk, R.C. Armstrong, and O. Hassager, Dynamics of Polymeric liquids, Vol. 1, 2nd ed., John Wiley & Sons, New York, NY 1987.
4. G Astarita and G. Merrucci. Principles of Non-Newtonian Fluid mechanics, McGraw-Hill 1974.

**MA-6105** **ADVANCED COMPLEX ANALYSIS**

**Course Title: ADVANCED COMPLEX ANALYSIS**

**Course Code: MA-6105**

**Credit Hours: 3**

**Prerequisites: Complex Analysis**

**Course Contents:** Analytic functions, Analytic functions as mappings, The open mapping theorem, The maximum principle, Schwartz lemma, Convex functions and Hadamard’s three-circle theorem, Elementary properties of holomorphic functions, Harmonic functions, Poisson’s integral formula and Dirichlet’s problem, Conformal mapping theorem, Analytic continuation, Monodromy theorem, Riemann surfaces, Modular functions and Picard theorem, Product theorems, Elliptic functions, Non-isolated removability theorems, Prime number theorem.

**RECOMMENDED BOOKS:**

1. Lars Ahlfors., “Complex Analysis: An Introduction to the Theory of Analytic Functions of one Complex Variable”, McGraw-Hill Co, 1979.
2. John B. Conway., “Functions of One Complex Variables I & II”, Springer, 1995.
3. Serge Lang., “Complex Analysis”, Springer, 2008.
4. Jane P. Gilman and Irwin Kra and Rubi E., “Complex Analysis: In the Spirit of LipmanBers”,
5. Theodore Gamelin., “Complex Analysis”, Springer, 2001.

**MA-6106 ADVANCE FUZZY ALGEBRA BCK ALGEBRA**

**Course Title: ADVANCE FUZZY ALGEBRA BCK ALGEBRA**

**Course Code: MA-6106**

**Credit Hours: 3**

**Prerequisites: Algebra, Group Theory**

**Course Contents:** Definition of BCK algebra. Examples. General properties of BCK algebra. Commutative BCK algebra. Ideal theory of BCK algebras. Definition, types, and examples of ideals in BCK algebra. Self-maps of BCK algebra. Types of ideals. Definition of self-maps. Right and left self-maps. Left regular maps and their general properties. Kernels and annihilators in BCK Algebras. Definition of kernel, annihilator, and related theorems.

**BCI Algebra**

**Classification of BCI Algebras:**

Implicative, Positive implicative BCI algebras.

**Classification and characterization of ideals in BCI Algebras:**

Ideals in BCI algebra, Strong and weak ideals. Characterization of BCI-algebras of order 6 with BCK part of order 1, 2, 3, 4 and 5. Fuzzy p-ideals, fuzzy h-ideals, fuzzy α-ideals, fuzzy BCI- (implicative, positive implicative, commutative) ideals and their interrelationship and their characterization based on extension principle and level sets. Intuitionistic fuzzy sets and their applications in ideal theory of BCK/BCI-algebras.

**Soft set theory**

Soft sets, definition, and examples from daily life. “Union”, “Extended intersection”, “Restricted Union”, “Restricted intersection” “AND” and “OR” operations defined on soft sets illustrated by various examples. Soft algebras, soft ideals, soft p-ideals, soft h-ideals, soft α-ideals and soft BCI- (implicative, positive implicative, commutative ideals). Applications of fuzzy sets to these soft ideals and detail discussion of their properties. Intuitionistic fuzzy soft sets, intuitionistic fuzzy soft ideals, and their properties. Decision making based on fuzzy soft sets and intuitionistic fuzzy soft sets.

**Fuzzy hyper structures**

Hyper structures, definition, and examples. Set valued functions. Applications of hyper structures in the ideal theory of BCK/BCI-algebras based on fuzzy sets. Fuzzy hyper BCK-algebras, fuzzy hyper BCI-Algebra, and their properties. Products and extensions of fuzzy hyper structures.

**RECOMMENDED BOOKS:**

1. Geoge G. Klir and Bo Yuan, Fuzzy sets, and Fuzzy Logic (Theory and Applications) (2010).
2. Meng, J. and Jun, Y.B.: BCK Algebras (Kyung Moon Sa. Co., Seoul., 1994).
3. Iseki, K. and Tanaka, S.: An Introduction to the Theory of BCK Algebra, Math. Japonica, 23, No.1(1978), 1-26.
4. Piergiulio Corsini and Violeta Laureano: Applications of Hyper structure theory (2003).

**MA-6107** **STACHASTIC PROCESS**

**Course Title: STACHASTIC PROCESS**

**Course Code: MA-6107**

**Credit Hours: 3**

**Prerequisites: Statistics, Probability, Linear Algebra**

**Course Contents: Basic Probability:**

Probability space and axioms, basic laws, conditional probability and Bayes rule, independence.

**Random Variables:**

Random variables, probability mass function (pmf), cumulative distribution function (cdf), probability density function (pdf), discrete, continuous, and mixed random variables, functions of random variables, generation of random variables. Pairs of random variables, joint, marginal, and conditional distributions, maximum likelihood (ML) and maximum a posteriori probability (MAP) detection. Expectation, mean, variance, characteristic function, covariance and correlation, Markov and Chebyshev inequalities, Jensen’s inequality, conditional expectation. Minimum mean square error (MMSE) estimation, linear estimation, jointly Gaussian random variables.

**Random Vectors:**

Extension of cdf, pdf, and pmf to more than two random variables, independence and conditional independence, covariance matrix, Gaussian random vectors, linear estimation - vector case. Modes of convergence, laws of large numbers, central limit theorem.

**Stochastic Processes:**

Discrete-time and continuous-time random processes, memoryless, independent increment, Stochastic, Markov, and Gaussian random processes, point processes. Stationary processes, autocorrelation functions and power spectral density (psd), white noise, bandlimited processes.

**RECOMMENDED BOOKS:**

1. Introduction to Stochastic Processes (Dover Books on Mathematics) January 24, 2013, [Erhan Cinlar](https://www.amazon.com/s/ref=dp_byline_sr_book_1?ie=UTF8&text=Erhan+Cinlar&search-alias=books&field-author=Erhan+Cinlar&sort=relevancerank) (Author)
2. A First Course in Stochastic Processes, 2nd Edition by [Samuel Karlin](https://www.amazon.com/Samuel-Karlin/e/B001IQW6Y8/ref=dp_byline_cont_book_1) (Author), [Howard M.](https://www.amazon.com/Howard-M.-Taylor/e/B00288KQ5G/ref=dp_byline_cont_book_2) [Taylor](https://www.amazon.com/Howard-M.-Taylor/e/B00288KQ5G/ref=dp_byline_cont_book_2) .

**MA-6108** **MAGNETOHYDRODYNAMICS-II**

**Course Title: MAGNETOHYDRODYNAMICS-II**

**Course Code: MA-6108**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, Calculus**

**Course Contents:**

**Flow of Conducting Fluid Past Magnetized Bodies:** Flow of an ideal fluid past magnetizedbodies, Fluid of finite electrical conductivity flow past a magnetized body.

**Dynamo Theories:** Elsasser’s Theory, Bullard’s Theory, Earth’s field Turbulent motion anddissipation, vorticityanology.

**Ionized Gases:** Effects of molecular structure, Currents in a fully ionized gas, partially ionizedgases, interstellar fields, dissipation in hot and cool clouds.

**RECOMMENDED BOOKS:**

1. Gowling, T.G., Magnetohydrodynamics, Interscience Publishers, 1963.
2. Kulikowskiy, A.G., and Lyubimov, G.A., Magnetohydrodynamics, A.Wesley, 1965.
3. Alfven, H., and Falthammar, C., Cosmical Electrodynamics, Clarendon Press, 1965.
4. Akliezer, A.I., Plasma Electrodynamics, Pergamon Press, 1975.
5. Kendall, P.C., and Plumption, C., Magnetohydrodynamics.
6. Aderson, J.E. Magnetohydrodynamics Shock Waves, M.I.T. Press, Cambridge, 1963.

**MA-6109 NILPOTENT AND SOLUBLE GROUPS**

**Course Title: NILPOTENT AND SOLUBLE GROUPS**

**Course Code: MA-6109**

**Credit Hours: 3**

**Prerequisites: Algebra, group Theory**

**Course Contents:** Normal and Subnormal Series, Abelian and Central Series, Direct Products, Finitely Generated Abelian Groups, Splitting Theorems, Soluble and Nilpotent Groups, Commutators Subgroup, Derived Series, The Lower and Upper Central Series, Characterization of Finite Nilpotent Groups, Fitting Subgroup, Frattini Subgroup, Dedekind Groups, Super soluble Groups, Soluble Groups with Minimal Condition. Subnormal Subgroups, Minimal Condition on Subnormal Subgroups, The Subnormal Socle, the Wielandt Subgroup and Wielandt Series, T-Groups, Power Automorphisms, Structure and Construction of Finite Soluble T-Groups.

**RECOMMENDED BOOKS:**

1. Robinson, D.J.S., A Course in the Theory of Groups, Graduate Textes in Mathematics 80, Springer, New York, 1982.
2. Doerk, K. and Hawkes, T., Finite Soluble Groups, De Gruyter Expositions in Mathematics 4, Walter De Gruyter, Berlin, 1992.

**MA-6110** **LA-SEMIGROUPS**

**Course Title: LA-SEMIGROUPS**

**Course Code: MA-6110**

**Credit Hours: 3**

**Prerequisites: Group Theory, Linear Algebra**

**Course Contents:** LA-semigroups and basic results, Connection with other algebraic structures, Medial and exponential properties, LA-semigroups defined by commutative inverse semigroups, Homomorphism theorems for LA-semigroups, Abelian groups defined by LA-semigroups, Embedding theorem for LA-semigroups, Structural properties of LA-semigroups, LA-semigroups as a semilattice of LA- sub semigroups, Locally associative LA-semigroups, Relations on locally associative LA-semigroups, Maximal separative homomorphic images of locally associative LA-semigroups, Decomposition of locally associative LA-semigroups.

**RECOMMENDED BOOKS:**

1. Clifford, A.H., and G.B. Preston., The Algebraic Theory of Semigroups, Vols. I & II, Amer. Math. Soc. Surveys, 7, Providence, R.I, 1967.

**MA-6111** **NUMERICAL SOLUTIONS OF NON-LINEAR SYSTEM OF EQUATIONS AND ORDINARY DIFFERENTIAL EQUATIONS**

**Course Title: NUMERICAL SOLUTIONS OF NON-LINEAR SYSTEM OF**

**EQUATIONS AND ORDINARY DIFFERENTIAL EQUATIONS**

**Course Code: MA-6111**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, Numerical Analysis**

**Course Contents:** Iterative techniques in matrix algebra. Estimates and iterative refinement. Numerical solutions of nonlinear system of equations. Fixed points for functions of several variables. Newton’s method for systems. Error estimates for fixed point and Newton’s method. Differential & Difference Equations: Differential Equation Problems Differential Equation Theory, Difference Equation Problems, Difference Equation Theory.

Numerical Differential Equations methods: Euler’s method, Analysis of Euler Method, Generalizations of Euler’s method, Runge-Kutta methods: Order conditions, Runge-Kutta methods with Error Estimates, Stability of Implicit Runge-Kutta methods. Linear Multistep methods: Order of Linear Multistep Methods, Error and Error Growth, Order & Stability Barriers. General Linear Multistep Methods: Consistency, Stability and convergence, Stability of General Linear Methods, Hybrid Methods.

**RECOMMENDED BOOKS:**

1. J.C. Butcher, “Numerical Methods for Ordinary Differential Equations”, John Wiley & Sons, 2008.
2. Burden, R. L. and Faires, J. D., (6th Ed.), “Numerical Analysis”, Brooks Cole Publishing Company, 1997.
3. Leon Lapidus& John H. Seinfeld, “Numerical Solutions of Ordinary Differential Equations”, Academic Press, 1971.
4. J.D. Lambort, “Numerical Methods for Ordinary Differential Systems”, Wiley, 1991.
5. AriehIserles, “A First Course in the Numerical Analysis of Differential Equations”, Cambridge University Press, 1996.

**MA-6112** **NUMERICAL SOLUTIONS OF INTEGRAL EQUATIONS**

**Course Title: NUMERICAL SOLUTIONS OF INTEGRAL EQUATIONS**

**Course Code: MA-6112**

**Credit Hours: 3**

**Prerequisites: Integral Equations, Numerical Analysis**

**Course Contents:** Linear integral equations of first & second kinds, solution of integral equations of seconds kind by successive substitutions. The Fredholm theory and its applications, Hilbert, Schmidt Theory of Integral Equations.

**RECOMMENDED BOOKS:**

1. Lovitt, W.V., “Linear Integral Equations”, Dover Publications, 1950.
2. Burden, R.L. and Fairs, J.D., “Numerical Analysis”, (Sixth edition), Brooks/Cole Publishing Co, 1997.
3. Mikhlin, S.G., “Integral Equations”, Taylor & Francis, 1961.
4. Tricomic, F.G., “Integral Equations”, Dover Publications, 1985.
5. Mathews, J.H., “Numerical Methods for mathematics, science and Engineering”, (Second edition), Prentice Hall, 1992.

**MA-6113 APPLIED LINEAR ALGEBRA-II**

**Course Title: APPLIED LINEAR ALGEBRA-II**

**Course Code: MA-6113**

**Credit Hours: 3**

**Prerequisites: Linear Algebra**

**Course Contents:** Adjoint operators; Unitary operators; Normal operators; Sesqui-linear form; Principal axis theorem; Positive forms; Properties of normal operators; Symmetric bilinear forms; Skew-symmetric bilinear forms; Orthogonal group; Pseudo-orthogonal group; Cyclic subspaces and annihilators; Companion matrix; Cyclic decomposition theorem; Rational form of a matrix; Jordan canonical form of a matrix; Hermitian forms and the spectral theorem; Spectral theorem for normal operators.

**RECOMMENDED BOOKS:**

1. Apostol, T.M., “Linear Algebra”, Wiley International, 1997.
2. Hoffman, K.M. and Kunze, R., (2nd Ed.), “Linear Algebra”, Prentice-Hall, 1971.
3. Karamat H. Dar, “Linear Algebra”, Carvan Book House-Lahore, 2007.
4. Lipschutz, S., “Schaum’s Outline of Linear Algebra”, McGraw Hill, 2002.
5. Mey, c.D., “Matrix Algebra and Applied Linear Algebra”, SIAM Publications, 2001.
6. Strang, G., “Introduction to Linear Algebra”, Wellesley-Cambridge Press, 1998.

**MA-6114 APPLIED FUNCTIONAL ANALYSIS-II**

**Course Title: APPLIED FUNCTIONAL ANALYSIS-II**

**Course Code: MA-6114**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, real Analysis**

**Course Contents:** Applications to bounded linear functionals to the space of real-valued continuous functions defined on [*a*,*b*]; Weak convergence; Weak completeness; Convergence of sequences of operators and functionals; Toeplitz limit theorem; Cesaro’s summability method; Holder’s summability method; Euler’s method. Weirstrass approximation theorem; Polya convergence theorem; Steklov’s theorem; Error estimates; Hellinger-Toeplitz theorem. Application of Banach’s theorem to linear equations, differential equations, and integral equations; Unbounded linear operators in quantum mechanics.

**RECOMMENDED BOOKS:**

1. F.Riesz and Nagy, “Functional Analysis”, Frederick unger publishing Co, 1965.
2. E. Kreyszig, “Introductory Functional Analysis with Applications”, John Wiley, and Sons, New York, 1989.
3. A.E. Taylor, “Introduction to Functional Analysis”, wiley International Edition, New York, 1957.
4. W. Rudin., “Functional Analysis”, McGraw-Hill, Inc., New York, 1991.
5. M.T. Nair., “Functional Analysis”, Prentice Hall of India, New Delhi, 2002.

**MA-6115** **ADVANCED OPERATIONS RESEARCH - II**

**Course Title: ADVANCED OPERATIONS RESEARCH-II**

**Course Code: MA-6115**

**Credit Hours: 3**

**Prerequisites: Linear Algebra**

**Course Contents:** Single variable optimization: examples and methods. Examples of optimisations in n-variable. n-variable unconstrained optimisation. Direct search methods: univariate, Nelder& Mead simplex. Line search methods: Steepest Descent, Wolfe’s, Armijo. Derivative methods: Newton and Quasi-Newton. Equality constrained optimisation. Penalty Function methods. Inequality constrained optimisation. Interior point and Barrier methods.

**RECOMMENDED BOOKS**

1. Dimitri P. Bertsekas, “Nonlinear Programming”, 2nd ed., Athena Scientific, 1999.
2. Singiresu S. Rao, “Engineering Optimization”, 4thed, John Wiley & Sons, 2009.
3. Michael Bartholomew-Biggs, “Nonlinear Optimization with Engineering Applications”, Springer, 2008.
4. Jorge Nocedal& Stephen J. Wright, “Numerical Optimization”, 2nd Ed., Springer, 2006.
5. Gionni Di Pillo& Fabio Schoen, “Nonlinear Optimization”, Springer, 2007.

**MA-6116** **TIME SCALE CALCULUS**

**Course Title: TIME SCALE CALCULUS**

**Course Code: MA-6116**

**Credit Hours: 3**

**Prerequisites: Linear Algebra**

**Course Contents:** Basic Definitions on Time Scales, Some Dynamic Equations, Nabla Dynamic equations, Reimann and Lebesgue integration, delta and Nabla integrals, Inequalities on time scales.

**RECOMMENDED BOOKS:**

1. Martin Bohner & Allan Peterson, “Advances in Dynamic Equations in Time Scales”, Birkhausar Boston, 2003.
2. R. P. Agarwal, “Difference equations and Inequalities”,Marcel Dekker, INC.,NewYork, 1992.
3. Martin Bohner & Allan Peterson, “Dynamic Equations on Time Scales, An Introduction with Applications”, Birkhausar Boston, 2001.

**MA-6117** **ALGEBRAIC TOPOLOGY-II**

**Course Title: ALGEBRAIC TOPOLOGY-II**

**Course Code: MA-6117**

**Credit Hours: 3**

**Prerequisites: Algebra & Topology**

**Course Contents: Homology Theory:** Delta-Complexes, Chain Complexes and Homology Groups, Simplicial Homology. Singular Homology, Homotopy Invariance, The Equivalence of Simplicial and Singular Homology, Homology of a Point, H0(X) for Path-wise Connected Spaces, H1(X) versus π1(X), Homology of Convex Subspaces of Rn , Chain Maps and Chain Homotopy, Homotopy Invariance, Homology of Contractible Spaces, Exact Sequences and Excision, Mayer-Vietoris Sequence, Homology of Spheres, No-Retraction and Brouwer Fixed Point Theorems, Relative Homology, Brouwer Degree for Maps Sn → Sn , Degree of the Antipodal Map

**Cohomology Theory:** Cohomology Groups: The Universal Coefficient Theorem, Cohomology of Spaces, Cup Product: The Cohomology Ring. A Kunneth Formula. Spaces with Polynomial Cohomology, Poincare Duality: Orientations and Homology, The Duality Theorem, Cup Product and Duality, Additional Topics: The Universal Coefficient Theorem for Homology, The General Kunneth Formula, H-Spaces and Hopf Algebras, The Cohomology of SO(n), Bockstein Homomorphisms, More About Ext. Transfer Homomorphisms, Local Coefficients

**RECOMMENDED BOOKS:**

1. Algebraic Topology, Allen Hatcher, *Cambridge University Press* (2002)
2. A Basic Course in Algebraic Topology, W. Massey, *Springer-Verlag* (1993).

**MA-6118** **GALOIS THEORY-II**

**Course Title: GALOIS THEORY-II**

**Course Code: MA-6118**

**Credit Hours: 3**

**Prerequisites: Algebra & group Theory**

**Course Contents:** Infinite Galois extensions, Topological groups, Krull Topology, Closed Subgroups, Inverse limits, Galois group and profinite groups, Fundamental theorem of infinite Galois Theory, Compactness of Galois group.

**RECOMMENDED BOOKS:**

1. Michael D. Fried and Moshe Jarden*, Field Arithmetic*, *3rd ed.*, Springer, Berlin, Heidelberg, 2008.
2. N. Jacobson, *Lectures in Abstract Algebra III*. *Theory of Fields and Galois Theory*, D. van Nostrand Company, Inc., Princeton, New York, 1964.

**MA-6119** **CONVEX FUNCTIONS**

**Course Title: CONVEX FUNCTIONS**

**Course Code: MA-6119**

**Credit Hours: 3**

**Prerequisites: Calculus, Real Analysis**

**Course Contents:** Convex functions on interval, young’s inequality and its consequences, Jensen’s Inequality, smoothness properties, Integral form of Jensen’s inequality, Hermite-Hadamard inequality, Algebraic version, gamma & beta functions, Multiplicative convexity, Am-GM inequality, (M, N)-convex functions, convex functions on normed linear spaces, convex functions in higher dimensions, continuity & Differentiability of convex functions.

**RECOMMENDED BOOKS:**

1. Constantin P. Niculescu, Lars-Erik Persson, CONVEX FUNCTIONS AND THEIR APPLICATIONS A contemporary approach, Springer 2004.
2. E. M. Alfsen, Compact convex sets and boundary integrals, Springer-Verlag, Berlin, 1971.
3. V. Barbu and Th. Precupanu, Convexity and Optimization in Banach Spaces, Ed. academies, Bucharest, and D. Reidel Publ. Co., Dordrecht, 1986.
4. J. M. Brewin and A. S. Lewis, Convex Analysis and Nonlinear Optimization. Theory and Examples., Springer-Verlag, Berlin, 2000.

**MA-6120** **APPROXIMATION THEORY**

**Course Title: APPROXIMATION THEORY**

**Course Code: MA-6120**

**Credit Hours: 3**

**Prerequisites: Linear Algebra, Numerical Analysis**

**Course Contents:** The approximation problem and existence of best approximations. The uniqueness of best approximations. Approximation operators. Polynomial interpolation. Uniform convergence of polynomial approximations. Least squares approximations. Properties of orthogonal polynomials, Order of convergence of polynomial approximations. Interpolation by piecewise polynomials. Chebyshev polynomials.

**RECOMMENDED BOOKS:**

1. Powell, M.J.D., (1988), “Approximation Theory and Methods”, Cambridge university Press.
2. Cheney, E.W., (2002), “Introduction to approximation Theory”, AMS Publications.
3. Achieser, N.I. and Akhiezer, N.I., (1992) “Theory of Approximation”, Dover publications.
4. Jackson, D., (2002), (2nd Ed.), “Introduction to Approximation Theory”, AMS Publications.
5. Phillips, G.M., (2003), “Interpolation and Approximation by Polynomials”, springer-Verlag.

**MA-6121** **RIEMANNIAN GEOMETRY**

**Course Title: RIEMANNIAN GEOMETRY**

**Course Code: MA-6121**

**Credit Hours: 3**

**Prerequisites: Differential Geometry, Topology**

**Course Contents:** Definition and examples of manifolds. Differential maps. Submanifolds. Tangents. Coordinate vector fields. Tangent spaces. Dual spaces. Multilinear functions. Algebra of tensors. Vector fields. Tensor fields. Integral curves. Flows. Lie derivatives. Brackets. Differential forms. Introduction to integration theory on manifolds. Riemannian and semi-Riemannian metrics. Flat spaces. Affine connection. Parallel translations. Covariant differentiation of tensor fields. Curvature and Torsion tensors. Connection of a semi-Riemannian tensor. Killing equation and killing vector fields. Geodesics. Conformal transformations and the Weyl tensor.

**RECOMMENDED BOOKS:**

1. Bishop, R.L. a nd Goldberg, S.I.: Tensor Analysis on Manifolds (Dover

publication, Inc. N.Y. 1980).

1. Do Carmo, M.P.: Riemannian Geometry (Birkhauser, Boston, 1992).
2. Lovelock, D. and Rund, E. F.: Differential Forms and Variational Principles (John-Wiley, 1975).
3. Langwitz, D.: Differential and Riemannian Geometry (Academic Press, 1970).
4. Abraham, R., Marsden, J.E. and Ratiu, T.: Manifolds, Tensor Analysis and Applications (Addison Wesley, 1983).

**MA-6122 GENERAL RELATIVITY-II**

**Course Title: GENERAL RELATIVITY-II**

**Course Code: MA-6122**

**Credit Hours: 3**

**Prerequisites: linear Algebra**

**Course Contents:** Black holes. Coordinate and essential singularities. Horizons. Coordinates passing through horizons. The Kruskal and the Carter-Penrose (CP) diagrams for the Schwarzschild geometry. The maximal ex tension. The Einstein-Rosen bridge. Wormholes. The CP diagram for the RN metric. The no-hair and cosmic censorship hypotheses. Gravitational forces about black holes. Black hole thermodynamics. Observational status and central black holes. Kaluza-Klein theory. Problems of quantum gravity. Quantization in curved space backgrounds and Hawking radiation. Isometries. Homotheties and their significance in Relativity.

**RECOMMENDED BOOKS:**

1. Wald, R.M.: General Relativity (The University of Chicago Press, 1984).
2. Stephani, Hans: General Relativity: An Introduction to the Theory of Gravitational Field (Cambridge University Press, 1990).
3. Misner, C.W., Thorne, K.S. and Wheeler, J.A.: Gravitation (W. H. Freeman and Co., 1973).
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