**COURSE CURRICULUM**

**2023-ONWARD**

**FOR**

 **BS Mathematics**



**DEPARTMENT OF MATHEMATICAL SCIENCES**

**FACULTY OF BASIC SCIENCES & HUMANITIES**

**UNIVERSITY OF ENGINEERING & TECHNOLOGY, TAXILA**

**January 2024**

**Title of the Course**: Ideology and Constitution of Pakistan

**Course Code:** PS-111
**Credit Hours:** 2+0

**Course Outline:**

**Introduction to the Ideology of Pakistan:**

Definition and significance of ideology.

Historical context of the creation of Pakistan (with emphasis on socio-political. religious, and cultural dynamics of British India between 1857 till 1947).

Contributions of founding fathers of Pakistan in the freedom movement including but not limited to Allama Muhammad Iqbal, Muhammad Ali Jinnah., etc.

Contributions of women and students in the freedom movement for separate homeland for Muslims of British India.

**Two-Nation Theory:**

Evolution of the Two-Nation Theory (Urdu-Hindi controversy, Partition of Bengal, Simla Deputation 1906, Allama lqbal's Presidential Address 1930, Congress Ministries 1937 Lahore Resolution 1940).

Role olcommunalism and religious differences.

**Introduction to the Constitution of Pakistan:**

Definition and importance of a constitution.

Ideological factors that shaped the Constitution(s) of Pakistan (Objectives Resolution 1949).

Overview of constitutional developments in Pakistan.

**Constitution and State Structure:**

Structure of Government (executive, legislature, and judiciary).

Distribution of powers between federal and provincial governments.

18th Amendment and its impact on federalism.

**Fundamental Rights, Principles of Policy and Responsibilities:**

Overview of fundamental rights guaranteed to citizens by the Constitution of Pakistan 1973 (Articles 8-28).

Overview of Principles of Policy (Articles 29-40).

Responsibilities of the Pakistani citizens (Article 5).

**Constitutional Amendments:**

Procedures for amending the Constitution.

Notable constitutional amendments and their implications.

**Suggested Instructional / Reading Materials**

1. "The Idea of Pakistan" by Stephen P. Cohen.

2. "Ideology of Pakistan" by Javed Iqbal.

3. "The Struggle for Pakistan" by 1.H. Qureshi.

4. "Pakistan the Formative Phase" by Khalid Bin Sayeed.

5. "Pakistan: Political Roots and Development" by Safdar Mahmood.

6. "Ideology of Pakistan" by Sharif-ul-Mujahid.

7."The Struggle for Pakistan: A Muslim Homeland and Global Politics" by Ayesha Jalal.

8."Jinnah, Pakistan and Islamic Identity: The Search for Saladin" by Akbar S. Ahmed.

9."The Making of Pakistan: A Study in Nationalism" by K.K. Aziz.

10. "Pakistan: A New History" by Ian Talbot.

11. "Pakistan in the Twentieth Century: A Political History" by Lawrence Ziring.

12. "The.Constitution of Pakistan 1973". Original.

13. "Constitutional and Political Development of Pakistan" by Hamid Khan.

14. "The Parliament of Pakistan" by Mahboob Hussain.

15. "Constitutional Development in Pakistan " by G.W. Choudhury.

16. "Constitution-Making in Pakistan: The Dynamics of Political Order" by G.W. Choudhury.

**Title of the Course**: Functional English

**Course Code:** ENG-111

**Credit Hours:** 2+0

**Course Outline:**

**Foundations of Functional English:**

Vocabulary building (contextual usage, synonyms, antonyms and idiomatic expressions)

Communicative grammar (subject-verb-agreement, verb tenses, fragments, run-ons,

modifiers, articles, word classes, etc.)

Word formation (affixation, compounding, clipping, back formation, etc.)

Sentence structure (simple, compound, complex and compound-complex)

Sound production and pronunciation

**Comprehension and Analysis:**

Understanding purpose, audience and context

Contextual interpretation (tones, biases, stereotypes, assumptions, inferences, etc.)

Reading strategies (skimming, scanning, SQ4R, critical reading, etc.)

Active listening (overcoming listening barriers, focused listening, etc.)

**Effective Communication:**

Principles of communication (clarity, coherence, conciseness, courteousness, correctness, etc.)

Structuring documents (introduction, body, conclusion and formatting)

Inclusivity in communication (gender-neutral language, stereotypes, cross-cultural

communication, etc.)

Public speaking (overcoming stage fright, voice modulation and body language)

Presentation skills (organization content, visual aids and engaging the audience)

Informal communication (small talk, networking and conversational skills)

Professional writing (business e-mails, memos, reports, formal letters, etc.)

**Suggested Practical Activities (Optional)**

As part of the overall learning requirements, students will also be exposed to relevant simulations, role-
plays and real-life scenarios and will be required to apply skills acquired throughout the course in the form of a final project.

**Suggested Instructional / Reading Materials:**

1. "Understanding and Using English Grammar" by Betty Schrampfer Azar.
2. "English Grammar in Use" by Raymond Murphy.
3. "The Blue Book of Grammar and Punctuation" by Jane Straus.
4. "English for Specific Purposes: A Learning-Centered Approach" by Tom Hutchinson and Alan

Waters.

1. "Cambridge English for Job-hunting" by Colm Downes.
2. "Practical English Usage" by Michael Swan.
3. "Reading Literature and Writing Argument" by Missy James and Alan P. Merickel.
4. "Improving Reading: Strategies, Resources, and Common Core Connections" by Jerry Johns

and Susan Lenski.

1. "Comprehension: A Paradigm for Cognition" by Walter Kintsch.
2. "Communication Skills for Business Professionals" by J.P. Verma and Meenakshi Raman.

**Title of the Course**: Computing Fundamentals

**Course Code:** CP-101

**Credit Hours:** 2+1

**Course Outline:**

**Information Processing in Digital Computers:** Evolution of Computer Hardware/Software:how the Computer System work, evolution of computer software through different generations, evolution of hardware in the light of Moore’s law. MODERN COMPUTERS**:** specifications of modern computers, Laptop, Mobiles, Tablets, Desktops, Servers, Gaming devices, Wearable devices, Digital Cameras,

**Hardware and Components:** Bits, Bytes, and Words and their relation tocomputer processor, conversion of the binary numbers to other bases (octal, decimal, and hexadecimal) and vice versa. Arithmetic, Logic, and Control Unit

**Programming Basics**: Pseudocode, Algorithm and Flowchart:How to write pseudocode, Understand algorithm to solve a computer problem, Represent an algorithm with the help of a flowchart. Low Level and High-level Programming:

Machine language, Assembly language, Assembly language of Intel processors, High level programming languages, C++, Java, Python. Software and its types:What is software, Application software, System softwar, Operating System, Types of Operating systems, Linux, Windows.

**Process and Memory Management: P**rocess, States of a Process, Process Management, Scheduling algorithms, Memory Hierarchy, Memory Management.

**Internet, communication and cyber security:** Internet, How internet works, IP addresses and domain names, Digital media on web (Graphics, Audio, Video, Plug ins). Security tools. Personal firewall. Antivirus. Malware. Internet and Network attacks.

**Careers:** IT department, Software and apps development, Technology sales service and repair, Technology education, Technology Consultation and Support.

**Lab Outline:** Basic machines organization including motherboard and peripheries, Networking devices, use of flow charts, introduction to various operating systems, introduction to office tools (Open Office: Ubuntu or variant based apps / MS Office), coding, executing and debugging simple programs, implementation of simple control structures, implementation of simple functions, implementation of different function styles, input/output, loops, conditional branching, graphics, effecting use of keyboard and introduction to Python.

**Suggested Instructional / Reading Materials:**

1. [Faithe Wempen](https://www.wiley.com/en-us/search?filters%5Bauthor%5D=Faithe+Wempen&pq=++), Computing Fundamentals: Introduction to Computers.
2. Andrew Tanenbaum, Data structures and algorithms
3. Pressman, Software Engineering: A Practitioner's Approach.

**Title of the Course**: Calculus and Analytic Geometry

**Course Code:** MTH-111

**Credit Hours:** 3+0

**Course Outline:**

**Introduction**

Real numbers and the real line, Functions, and their graphs, shifting graphs, Solution of equations involving absolute values, Inequalities, Complex numbers system, Polar form of complex number, De-Movier’s Theorem, Circular functions, Hyperbolic functions, and Logarithmic functions.

 **Limits and Continuity**

Rates of change and limits, Rules for finding limits, Formal Definition of limits, Extension of the limit concept, Continuity

 **Derivatives and its Applications**

Differentiable functions, Differentiation of polynomial, rational and transcendental functions, Mean value theorem and applications, Higher derivatives, Leibniz’s theorem, L’ Hospital’s rule, Intermediate value theorem, Rolle’s theorem, Taylor’s, and Maclaurin’s theorem with their remainders.

**Integration**

Techniques of evaluating indefinite integrals, Integration by substitutions, Integration by parts, Change of variable in indefinite integrals, Definite integrals, Fundamental theorem of calculus,

Reduction formulas for algebraic and trigonometric integrands, Improper integrals, Gamma functions.

 **Analytic Geometry of Two Dimensions**

 Conic section and quadratic equations, Classification of conic section by eccentricity, Translation and rotation of axis, Properties of circle, parabola, ellipse, hyperbola, Polar coordinates, conic sections in polar coordinates, Graphing in polar coordinates, Tangents and normal, pedal equations, parametric representations of curves

**Applications of Integration**

Asymptotes, Relative extrema, Points of inflections and concavity, Singular points, tangents at the origin, Graphing of Cartesian and polar curves, Area under the curve, area between two curves, Arc length and intrinsic equations, Curvature, radius and center of curvature, Involute and evolute, envelope.

**Analytic Geometry of Three Dimensions**

Rectangular coordinates system in a space, Cylindrical and spherical coordinate system, Direction ratios and direction cosines of a line, Equation of straight lines and planes in three dimensions, shortest distance between skew lines, Equation of sphere, cylinder, cone, ellipsoids, paraboloids, hyperboloids, Quadric and ruled surfaces, Spherical trigonometry, Direction of Qibla.

**Suggested Instructional / Reading Materials:**

1. Thomas, *Calculus*, 11th Edition. Addison Wesley Publishing Company, 2005.
2. H. Anton, I. Bevens, S. Davis, *Calculus*, 8th Edition, John Wiley & Sons, Inc. 2005.
3. C.H. Edward and E.D. Penney, *Calculus and Analytics Geometry*, Prentice Hall, Inc. 1988.
4. E. W. Swokowski, *Calculus and Analytic Geometry*, PWS Publishers, Boston, Massachosetts, 1983.
5. Frank A. Jr, Elliott Mendelson, *Calculus*, Schaum’s outlines series, 4th Edition, 1999.
6. Dennis G. Zill & Patric D. Shanahan, *Complex Analysis*, Jones & Barlett Publishers, 2003.

**Title of the Course:** Physics-I (Basic Mechanics)

**Course Code:** PHY-111

**Credit Hours:** 3+1

**Course Outline:**

**Basic Concepts:** Units and Dimensions, SI Units, Changing Units, Scalars and Vectors, Adding Vectors: Graphical as well as Component Method, Multiplying Vectors: Dot and Cross Products.

**Motion in One, Two and Three Dimensions:** Position & Displacement, Velocity and Acceleration, Motion under Constant Acceleration, Projectile Motion, Uniform Circular Motion, Relative Velocity and Acceleration in One and Two Dimensions, Inertial and Non-Inertial Reference Frames.

**Newton’s Laws:** Newton’s Laws of Motion and their Applications involving some particular forces including Weight, Normal Force, Tension, Friction, and Centripetal Force, Newton’s Law of Gravitation, Gravitational Potential Energy, Escape Velocity, Kepler’s Laws, Satellite Orbits & Energy.

**Work and Kinetic Energy:** Work done by Constant and Variable Forces: Gravitational and Spring Forces, Power, Conservative and Non-conservative Forces, Work and Potential Energy, Isolated Systems and Conservation of Mechanical Energy, Work Done by External Forces including Friction and Conservation of Energy.

**System of Particles:** Motion of a System of Particles and Extended Rigid Bodies, Center of Mass and Newton’s Laws for a System of Particles, Linear Momentum, Impulse, Momentum & Kinetic Energy in One and Two Dimensional Elastic and Inelastic Collisions.

**Rotational Motion:** Rotation about a Fixed Axis, Angular Position, Angular Displacement, Angular Velocity and Angular Acceleration, Rotation under Constant Angular Acceleration, relationship between Linear and Angular Variables, Rotational Inertia, Parallel-axis Theorem, Torque and Newton’s Law for Rotation, Work and Rotational Kinetic Energy, Power, Rolling Motion, Angular Momentum for a single Particle and a System of Particles, Conservation of Angular Momentum, Precession of a Gyroscope, Static Equilibrium involving Forces and Torques, Determination of moment of inertia of various shapes i.e. for disc, bar and solid sphere.

**Angular Momentum:** Angular Velocity, Conservation of angular momentum, effects of Torque and its relation with angular momentum.

**Simple Harmonic Motion (SHM):** Amplitude, Phase, Angular Frequency, Velocity and Acceleration in SHM, Linear and Angular Simple Harmonic Oscillators, Energy in SHM, Simple Pendulum, Physical Pendulum, SHM and Uniform Circular Motion, Damped Harmonic Oscillator.

**Special Theory of Relativity:** Inertial and non-inertial frame, Postulates of Relativity, The Lorentz Transformation, Derivation, Assumptions on which inverse transformation is derived, Consequences of Lorentz transformation, Relativity of time, Relativity of length, Relativity of mass, Transformation of velocity, variation of mass with velocity, mass energy relation and its importance, relativistic momentum and Relativistic energy, (Lorentz invariants) $ E^{2}=c^{2}p^{2}+m\_{°}^{2}c^{4}$

**Suggested Instructional / Reading Materials:**

1. Halliday, R. Resnick and J. Walker, “Fundamentals of Physics”, John Wiley & Sons, 9th ed. 2010.
2. R. A. Serway and J. W. Jewett, “Physics for Scientists and Engineers”, Golden Sunburst Series, 8thed. 2010.
3. R. A. Freedman, H. D. Young, and A. L. Ford (Sears and Zeemansky),“University Physics with Modern Physics”, Addison-Wesley-Longman,13th International ed. 2010.
4. F. J Keller, W. E. Gettys and M. J. Skove, “Physics: Classical and Modern, McGraw Hill. 2nd ed. 1992.
5. D. C. Giancoli, “Physics for Scientists and Engineers, with Modern Physics”, Addison-Wesley, 4th ed. 2008.

**Title of the Course**: Foundation of Mathematics

**Course Code:** MTH-112

**Credit Hours:** 3+0

**Course Outline:**

[**Introduction to the foundations of mathematics**](http://settheory.net/foundations/introduction)

[Variables, sets, functions and operations](http://settheory.net/foundations/variables-sets), [Form of theories](http://settheory.net/foundations/theories)

**Set Theory**; Axioms, Set Generation, Currying & Tupples, Uniquness quantifiers, Injection, Bijection, Philosophical aspects : [Time in set theory](http://settheory.net/sets/time-in-set-theory), [Interpretation of classes](http://settheory.net/sets/classes2), [Concepts of truth in mathematics](http://settheory.net/sets/truth)

**Algebra** : [Galois connections](http://settheory.net/algebra/galois-connection), [Relational systems and concrete categories](http://settheory.net/algebra/concrete-categories) , [Algebras](http://settheory.net/algebra/algebras), [Special morphisms](http://settheory.net/algebra/morphisms), [Monoids and categories](http://settheory.net/algebra/monoid), [Actions of monoids and categories](http://settheory.net/algebra/monoid-actions), [Invertibility and groups](http://settheory.net/algebra/group), [Properties in categories](http://settheory.net/algebra/category), [Initial and final objects](http://settheory.net/algebra/initial-final), [Products of systems](http://settheory.net/algebra/products), [Basis](http://settheory.net/algebra/basis), [Composition of relations](http://settheory.net/algebra/relations)

**Arithmetic**: [Algebraic terms](http://settheory.net/arithmetic/terms), [Quotient systems](http://settheory.net/arithmetic/quotient), [Term algebras](http://settheory.net/arithmetic/term-algebra), [Integers and recursion](http://settheory.net/arithmetic/integers), [Presburger Arithmetic](http://settheory.net/arithmetic/Presburger), [Finiteness](http://settheory.net/arithmetic/finiteness), [Countability and Completeness](http://settheory.net/arithmetic/completeness), [More recursion tools](http://settheory.net/arithmetic/recursion), [Non-standard models of Arithmetic](http://settheory.net/arithmetic/non-standard-arithmetic), [Developing theories : definitions](http://settheory.net/arithmetic/development): [Constructions](http://settheory.net/arithmetic/constructions). [The Berry paradox](http://settheory.net/arithmetic/Berry-paradox)

**Suggested Instructional / Reading Materials:**

1. Kenneth Kunen, The Foundations of Mathematics, 2007.
2. D. Marker Model Theory, An Introduction, Springer-Verlag, 2002.
3. D. Varberg, E. J. Purcell, and S. E. Rigdon, Calculus, Instructor’s Edition, 8th,edition, Prentice Hall, 2000.
4. K. Kunen, Set Theory, North-Holland Pub. Co., 1980.

**Title of the Course:** Islamic Studies/Ethics

**Course Code:** ISL-121

**Credit Hours:** 2+0

**Course Outline:**

**Introduction to Islam:**

Definition of Islam and its core beliefs.

The Holy Quran (introduction, revelation and compilation).

Hadith and Sunnah (compilation, classification, and significance).

Key theological concepts and themes (Tawhid, Prophethood, Akhirah etc.).

Sirah of the Holy Prophet (Peace Be Upon Him) as Uswa-i-Hasana:

Life and legacy of the Holy Prophet PBUH.

Diverse roles of the Holy Prophet PBUH (as an individual, educator, peace maker, leader etc.).

**Islamic History and Civilization:**

World before Islam.

The Rashidun Caliphate and expansion of Islamic rule.

Contribution of Muslim scientists and philosophers in shaping world civilization.

**Islamic Jurisprudence (Filth):**

Fundamental sources of Islamic jurisprudence.

Pillars of Islam and their significance.

Major schools of Islamic jurisprudence.

Significance and principles of Ijtihad.

**Family and Society in Islam:**

Status and rights of women in Islamic teachings.

Marriage, family, and gender roles in Muslim society.

Family structure and values in Muslim society.

**Islam and the Modern World:**

Relevance of Islam in the modern world (globalization, challenges and prospects).

Islamophobia, interfaith dialogue, and multiculturalism.

Islamic viewpoint towards socio-cultural and technological changes.

**Suggested Instructional / Reading Materials**

1. "The Five Pillars of Islam: A Journey Through the Divine Acts of Worship" by Muhammad Mustafa Al-Azami.
2. "The Five Pillars of Islam: A Framework for Islamic Values and Character Building" by Musharraf Hussain.
3. "Towards Understanding Islam" by Abul A' la Mawdudi.
4. "Islami Nazria e Hayat" by Khurshid Ahmad.
5. "An Introduction to Islamic Theology" by John Renard.
6. "Islamic Civilization Foundations Belief & Principles" by Abul A' la Mawdudi.
7. "Women and Social Justice: An Islamic Paradigm" by Dr. Anis Ahmad.
8. "Islam: Its Meaning and Message" by Khurshid Ahmad.

**Title of the Course:** Physics-II (Electricity and Magnetism)

**Course Code:** PHY-121

**Credit Hours:** 3+1

**Course Outline:**

**Electrostatics:** Electric Charge, Conductors and Insulators, Coulomb’s Law, Electric Fields due to a Point Charge and an Electric Dipole, Electric Field due to a Charge Distribution, Electric Dipole in an Electric Field, Electric Flux, Gauss’ Law and its Applications in Planar, Spherical and Cylindrical Symmetry.

**Electric Potential**: Equipotential Surfaces, Potential due to a Point Charge and a Group of Point Charges, Potential due to an Electric Dipole, Potential due to a Charge Distribution, Relation between Electric Field and, Electric Potential Energy.

**Capacitors and Capacitance:** Parallel Plate, Cylindrical and Spherical capacitors, Capacitors in Series and Parallel, Energy Stored in an Electric Field, Dielectrics and Gauss’ Law.

**DC Circuits:** Electric Current and Current Density, Resistance and Resistivity, Ohm’s Law, Power in Electric Circuits, Semiconductors and

Superconductors, Work, Energy, and EMF, Resistances in Series and

Parallel, Single and Multi-loop Circuits, Kirchhoff’s Rules, RC Circuits, Charging and Discharging of a Capacitor.

**Magnetic Field and Magnetic Force:** Crossed Electric and Magnetic Field sand their Applications, Hall Effect, Magnetic Force on a Current Carrying Wire, Torque on a Current Loop, Magnetic Dipole Moment, Magnetic Field Due to a Current, Force between two Parallel Currents, Ampere’s Law, Biot-Savart Law: Magnetic Field due to a Current, Long Straight Wire carrying Current, Solenoids

and Toroids, A current-carrying Coil as a Magnetic Dipole, Inductance, Faraday’s Law of Induction, Lenz’s Law, Induction and Energy Transfers, Induced Electric Fields, Inductors and Inductances, Self Inductance, RL Circuits, Energy Stored in a Magnetic Field, Energy Density, Mutual Induction.

**Alternating Fields and Currents:** LC Oscillations, Damped Oscillations in an RLC circuit, Alternating Currents, Forced Oscillations, Resistive, Capacitive, and Inductive Loads, RLC series Circuit, Power in AC Circuits, Transformers, Gauss’ Law for Magnetism, Induced Magnetic Fields, Displacement Current, Spin & Orbital Magnetic Dipole Moment, Diamagnetism, Paramagnetism, Ferromagnetism, Hysteresis.

**Suggested Instructional / Reading Materials:**

1. D. Halliday, R. Resnick and J. Walker, “Fundamentals of Physics”, John Wiley & Sons, 9th ed. 2010.
2. R. A. Serway and J. W. Jewett, “Physics for Scientists and Engineers”, Golden Sunburst Series, 8thed. 2010.
3. R. A. Freedman, H. D. Young, and A. L. Ford (Sears and Zeemansky),“University Physics with Modern Physics”, Addison-Wesley-Longman,13th International ed. 2010.
4. F. J Keller, W. E. Gettys and M. J. Skove, “Physics: Classical and Modern, McGraw Hill. 2nd ed. 1992.
5. D. C. Giancoli, “Physics for Scientists and Engineers, with Modern Physics”, Addison-Wesley, 4th ed. 2008.

**Title of the Course**: Expository Writing

**Course Code:** ENG-121

**Credit Hours:** 3+0

**Course Outline:**

**Introduction to Expository Writing:**

Understanding expository writing (definition, types, purpose and applications)

Characteristics of effective expository writing (clarity, coherence and organization)

Introduction to paragraph writing

**The Writing Process:**

Pre-writing techniques (brainstorming, free-writing, mind-mapping, listing, questioning

and outlining etc.)

Drafting (three stage process of drafting techniques)

Revising and editing (ensuring correct grammar, clarity, coherence, conciseness etc.)

Proof reading (fine-tuning of the draft)

Peer review and feedback (providing and receiving critique)

**Essay Organization and Structure:**

Introduction and hook (engaging readers and introducing the topic)

Thesis statement (crafting a clear and focused central idea)

Body Paragraphs (topic sentences, supporting evidence and transitional devices)

Conclusion (types of concluding paragraphs and leaving an impact)

Ensuring cohesion and coherence (creating seamless connections between paragraphs)

**Different Types of Expository Writing:**

Description

Illustration

Classification

Cause and effect (exploring causal relationships and outcomes)

Process analysis (explaining step-by-step procedures)

Comparative analysis (analyzing similarities and differences)

**Writing for Specific Purposes and Audiences:**

Different types of purposes (to inform, to analyze, to persuade, to entertain etc.)

Writing for academic audiences (formality, objectivity, and academic conventions)

Writing for public audiences (engaging, informative and persuasive language)

Different tones and styles for specific purposes and audiences

**Ethical Considerations:**

Ensuring original writing (finding credible sources, evaluating information etc.)

Proper citation and referencing (APA, MLA, or other citation styles)

Integrating quotes and evidences (quoting, paraphrasing, and summarizing)

Avoiding plagiarism (ethical considerations and best practices)

**Suggested Practical Activities (Optional)**

As part of the overall learning requirements, students will be required to build a writing portfolio having a variety of expository texts and present the same at the end of the course showcasing proficiency in expository writing.

**Suggested Instructional / Reading Materials:**

1. "The St. Martin's Guide to Writing" by Rise B. Axelrod and Charles R. Cooper.
2. "They Say / I Say: The Moves That Matter in Academic Writing" by Gerald Graff and Cathy
3. Birkenstein.
4. "Writing Analytically" by David Rosenwasser and Jill Stephen.
5. 4."Style: Lessons in Clarity and Grace" by Joseph M. Williams and Joseph Bizup.
6. 5."The Elements of Style" by William Strunk Jr. and E.B. White.
7. 6."Good Reasons with Contemporary Arguments" by Lester Faigley and Jack Selzer.
8. 7. "Writing to Learn: How to Write - and Think - Clearly About Any Subject at All" by William
 Zinsser.
9. "The Norton Field Guide to Writing" by Richard Bullock, Maureen Daly Goggin, and Francine
10. Weinberg.
11. "The Art of Styling Sentences" by Ann Longknife and K.D. Sullivan.
12. "Writing Today" by Richard Johnson-Sheehan and Charles Paine.

**Title of the Course:** Computer Programming

**Course Code:** CP-107

**Credit Hours:** 3+1

**Course Outline:**

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| **Introduction to Programming:** Computer systems, Classes of computers, Hardware & Software, Computer languages, Compiler, Linker and Assembler, Algorithms, Program design process, Software life cycle, Layout of a simple c++ program, Testing and debugging**C++ Basics:** Variables (Declaration, initialization, scope), Assignment statements, Output using cout, Input using cin, Escape sequences, Data types, Arithmetic operators and operations, Comment, Naming constants**Conditional Statements:** Boolean expression, Logical operators (&&, ||, !), Comparison operators (==, !=, <, >, <=, >=), Precedence rules, Flow of control, Simple branching mechanism, Simple if , If-else, Multiway if-else, Nested if-else, Switch statement, If-else vs switch, Conditional operator, break statement, Block, Goto statement, Enumeration type, continue statement**Looping Statements:** postfix & prefix increment operator, postfix & prefix decrement operator, Loop mechanism, Types of loops, for loop, while loop, do while loop, Which loop to use, Infinite loop, Nested loop**Single & Multidimensional Arrays:** Arrays, Single dimensional arrays(Declaration & initialization), Displaying array, setw (manipulator), Array index out of bound, Arrays in memory, Using defined constant as array size, Multidimensional arrays (Declaration and initialization) **Structures:** Structures, Structure creation, Creating structure instances, Accessing structure members, Initializing structure**Functions:** C++ functions, Top-down method, Pre-defined functions, Programmer defined functions, Function declaration, Function definition, Function call, Return statement, Void functions, Passing arrays as function arguments, Parameters and arguments, Difference between call by reference and value, Call by reference parameter, Functions calling functions, Preconditions and postconditions, Local, global and block scope, Overloading a function name**Pointers:** Pointers, Pointer variable declaration, The \* and & operator, Use of assignment operator, The new operator, Basic memory management, The delete operator, Static and dynamic variables, Type definitions, Array as pointers, Dynamic arrays**Classes:** Introduction to classes, C++ Classes, The dot operator, Scope resolution operator, Constructor, Destructor, Array of objects, Objects as arguments, Returning objects from function, Inline functions in C++**Strings:** C++ strings, Null character, C-string declaration and initialization, Inputting string, getline, String manipulation, Predefined c-string functions, 2-D chart array, String class, String object declaration and initialization, String operations, String array, Conversion between string and numbers**File Handling:** File header files, Opening and closing files, Reading from file, Writing in file, Appending file, Using file operations**Computing Programming Lab:** Introduction to Visual Studio 2022 environment. Basics of C++. Branching statements (If, If-else, nested If-else, switch, goto) and enumerations. Looping statements. Break statement, continue statement and increment/decrement operator. Nested loops. Arrays (1D and 2D). Structures, Functions, and Pointers in C++. Passing arrays to function and function name overloading. Classes I&II. Strings in c++ (C-string and string class). File Handling. Project.**Suggested Instructional / Reading Materials:**  |
| 1. Walter Savitch, Addison Wesley, "Problem solving with C++", 9th Edition.
2. Deitel & Deitel, “C++ How to program”, Prentice Hall, Latest Edition
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**Title of the Course:** Linear Algebra-I

**Course Code:** MTH-121

**Credit Hours:** 3+0

**Course Outline:**

Euclidean spaces, Vector spaces, Subspaces, Linear independence and dependence, Basis and dimensions, Rank and nullity of matrices, Inner product spaces, Angle and Orthogonality in Inner product spaces, Orthogonal basis; Gram–Schmidt process. Orthogonal matrices, Change of basis, Eigenvalues and eigenvectors. Diagonalization, Orthogonal diagonalization.

Linear transformations, Linear functional and dual spaces, Kernel and range, Inverse linear transformation, Matrices of general linear transformation, Rank and nullity of linear transformation, Applications to relevant problems.

**Suggested Instructional / Reading Materials:**

1. Anton, H. (1994) “Elementary Linear Algebra” John Wiley publisher.
2. Donald, J. W. (1999) “Linear Algebra” McGraw–Hill.
3. Seymour, L. and Marc, L. (2001) “Linear Algebra” Schaum’s Outline Series, McGraw–Hill.
4. Bretscher, O. (1997) “Linear Algebra” Prentice–Hall.
5. Koleman, B. (1997) “Introductory Linear Algebra with Applications” (Sixth edition) Prentice Hall

**Title of the Course:** Applications of Information and Communication Technologies (ICT)

**Course Code:** ICT-211

**Credit Hours:** 2+0

**Outlines:**

**Introduction to Information and Communication Technologies:**

Components of Information and Communication Technologies (basics of hardware, software, ICT platforms, networks, local and cloud data storage etc.).

Scope of Information and Communication Technologies (use of ICT in education, business, governance, healthcare, digital media and entertainment, etc.).

Emerging technologies and future trends.

**Basic ICT Productivity Tools:**

Effective use of popular search engines (e.g., Google, Bing, etc.) to explore World Wide Web.

Formal communication tools and etiquettes (Gmail, Microsoft Outlook, etc.).

Microsoft Office Suites (Word, Excel, PowerPoint).

Google Workspace (Google Does, Sheets, Slides).

Dropbox (Cloud storage and file sharing), Google Drive (Cloud storage with Google Does integration) and Microsoft OneDrive (Cloud storage with Microsoft Office integration).

Evernote (Note-taking and organization applications) and OneNote (Microsoft's digital notebook for capturing and organizing ideas).

Video conferencing (Google Meet, Microsoft Teams, Zoom, etc.).

Social media applications (Linkedln, Facebook, Instagram, etc .).

**ICT in Education:**

Working with learning management systems (Moodie, Canvas, Google Classrooms, etc.).

Sources of online education courses (Coursera, edX , Udemy, Khan Academy, etc.).

Interactive multimedia and virtual classrooms.

**ICT in Health and Well-being:**

Health and fitness tracking devices and applications (Google Fit, Samsung Health, Apple Health, Xiaomi Mi Band, Runkeeper, etc.).

Telemedicine and online health consultations (OLADOC, Sehat Kahani, Marham, etc.).

**ICT in Personal Finance and Shopping:**

Online banking and financial management tools (JazzCash, Easypaisa, Zong PayMax, I LINK and MNET, Keenu Wallet, etc.).

E-commerce platforms (Daraz.pk, Telemart, Shophive, etc.)

**Digital Citizenship and Online Etiquette:**

Digital identity and online reputation.

Netiquette and respectful online communication.

Cyberbullying and online harassment.

**Ethical Considerations in Use of ICT Platforms and Tools:**

Intellectual property and copyright issues.

 Ensuring originality in content creation by avoiding plagiarism and unauthorized use of information sources.

Content accuracy and integrity (ensuring that the content shared through ICT platforms is free from misinformation, fake news, and manipulation).

**Practical Requirements**

As part of the overall learning requirements, the course will include:

1. Guided tutorials and exercises to ensure that students are proficient in commonly used software

applications such as word processing software (e.g., Microsoft Word), presentation software (e.g., Microsoft PowerPoint), spreadsheet software (e.g., Microsoft Excel) among such other tools. Students may be assigned practical tasks that require them to create documents, presentations, and spreadsheets etc.

2. Assigning of tasks that involve creating, managing, and organizing files and folders on both
local and cloud storage systems. Students will practice file naming conventions, creating
directories, and using cloud storage solutions (e.g., Google Drive, OneDrive).

3. The use of online learning management systems (LMS) where students can access course
materials, submit assignments, participate in discussion forums, and take quizzes or tests. This
will provide students with the practical experience with online platforms commonly used in
education and the workplace.

**Suggested Instructional / Reading Materials**

1. "Discovering Computers'' by Vermaat, Shaffer, and Freund.
2. "GO! with Microsoft Office" Series by Gaskin, Vargas, and McLellan.
3. "Exploring Microsoft Office" Series by Grauer and Poatsy.
4. "Computing Essentials" by Morley and Parker.
5. "Technology in Action" by Evans, Martin, and Poatsy.

**Title of the Course:** Physics-III (Heat and Thermodynamics)

**Course Code:** PHY-211

**Credit Hours:** 3+1

**Course Outline:**

**Basic Concepts and Definitions in Thermodynamics:** Thermodynamic system, Surrounding and Boundaries. Type of systems. Macroscopic and microscopic description of system. Properties and state of the substance: Extensive and Intensive properties, Equilibrium, Mechanical and Thermal Equilibrium. Processes and Cycles: Isothermal, Isobaric and Isochoric. Zeroth Law of Thermodynamics, Consequence of Zeroth law of Thermodynamics. The state of the system at Equilibrium.

**Heat and Temperature:** Temperature, Kinetic theory of ideal gas, Work done on an ideal gas, Review of previous concepts. Internal energy of an ideal gas: Equipartition of Energy, Intermolecular forces, Qualitative discussion, The Virial expansion, The Van der Waals equation of state.

**Thermodynamics:** First law of thermodynamics and its applications to adiabatic, isothermal, cyclic and free expansion. Reversible and irreversible processes. Second law of thermodynamics, Carnot theorem and Carnot engine. Heat engine, Refrigerators. Calculation of efficiency of heat engines. Thermodynamic temperature scale: Absolute zero, Entropy, Entropy in reversible process, Entropy in irreversible process. Entropy and Second law of thermodynamics, Entropy and Probability. Thermodynamic Functions: Thermodynamic functions (Internal energy, Enthalpy, Gibb’s functions, Entropy, Helmholtz functions), Maxwell’s relations, TdS equations, Energy equations and their applications. Low Temperature Physics, Joule-Thomson effect and its equations. Thermoelectricity: Thermocouple, Seabeck’s effect, Peltier’s effect, Thomson effect.

**Introduction to Statistical Mechanics:** Statistical distribution and mean values, Mean free path and microscopic calculations of mean free path. Distribution of Molecular Speeds, Distribution of Energies, Maxwell distribution, Maxwell Boltzmann energy distribution, Internal energy of an ideal gas, Brownian Motion Legvaian equation, Qualitative description.

**Suggested Instructional / Reading Materials:**

1. Technical Writing, Process and Product, Fifth Edition by Sharon J. Gerson and Steven.M. Gerson
2. Hand Book of Technical writing by Davis A. MacMurrey
3. Dissertation and Project Report How to Write by Kathleen Macmillan
4. Communication Skills by Leena Sen

**Title of the Course:** Multivariable Calculus

**Course Code:** MTH-211

**Credit Hours:** 3+0

**Course Outline:**

**Sequence and Series:**

Sequences, Infinite series, Convergence of sequence and series, The integral test, Comparison tests, Ratio test, Root test, Alternative series, Absolute and conditional convergence, Power series, Interval, and radius of convergence.

**Multivariable Functions and Partial Derivatives:**

Functions of two variables, Graphs of functions of two variables, Contour diagrams, Linear functions, Functions of three variables, Limit and continuity of a function of two variables, The partial derivative, Computing partial derivatives algebraically, The second-order partial derivative, Local linearity and the differential, Tangent planes and normal lines, Optimization, Maxima and minima of a function of two variables, Lagrange multipliers, Various methods for finding area and volume surface of revolution.

**Multiple Integrals:**

Double integral in rectangular and polar form, Triple integral in rectangular, Cylindrical, and spherical coordinates, Substitutions in multiple integrals, Areas, Moments and Centers of mass.

**Suggested Instructional / Reading Materials:**

1. Thomas, *Calculus*, 11th Edition. Addison Wesley Publishing Company, 2005.
2. H. Anton, I. Bevens, S. Davis, *Calculus*, 8th Edition, John Wiley & Sons, Inc. 2005.
3. C.H. Edward and E.D. Penney, *Calculus and Analytics Geometry*, Prentice Hall, Inc. 1988.
4. E. W. Swokowski, *Calculus and Analytic Geometry*, PWS Publishers, Boston, Massachosetts, 1983.
5. Frank A. Jr, Elliott Mendelson, *Calculus*, Schaum’s outlines series, 4th Edition, 1999.
6. Hughes-Hallet, Gleason, McCalum, *et al.*, *Calculus Single and Multivarible*, 3rd Edition John Wiley & Sons, Inc 2002.

**Title of the Course**: Quantitative Reasoning-I

**Course Code:** QR-211

**Credit Hours**: 3+0

**Course Outline**:

**Numerical Literacy**

Number system and basic arithmetic operations;

Units and their conversions, dimensions, area, perimeter and volume;

Rates, ratios, proportions and percentages;

Types and sources of data;

Measurement scales;

Tabular and graphical presentation of data;

Quantitative reasoning exercises using number knowledge.

**Fundamental Mathematical Concepts**

Basics of geometry (lines, angles, circles, polygons etc.);

Sets and their operations;

Relations, functions, and their graphs;

Exponents, factoring and simplifying algebraic expressions;

Algebraic and graphical solutions of linear and quadratic equations and inequalities;

Quantitative reasoning exercises using fundamental mathematical concepts.

**Fundamental Statistical Concepts**

Population and sample;

Measures of central tendency, dispersion and data interpretation;

Rules of counting (multiplicative, permutation and combination);

Basic probability theory;

Introduction to random variables and their probability distributions;

Quantitative reasoning exercises using fundamental statistical concepts.

**Suggested Instructional / Reading Materials**

1. "Quantitative Reasoning: Tools for Today's Informed Citizen" by Bernard L. Madison, Lynn and Arthur Steen.

2. "Quantitative Reasoning for the Information Age" by Bernard L. Madison and David M. Bressoud.

3. "Fundamentals of Mathematics" by Wade Ellis.

4. "Quantitative Reasoning: Thinking in Numbers" by Eric Zaslow.

5. "Thinking Clearly with Data: A Guide to Quantitative Reasoning and Analysis" by Ethan Bueno de Mesquita and Anthony Fowler.

6. "Using and Understanding Mathematics: A Quantitative Reasoning Approach" by Bennett, J. 0., Briggs, W. L., & Badalamenti, A.

7. "Discrete Mathematics and its Applications" by Kenneth H. Rosen.

8. "Statistics for Technology: A Course in Applied Statistics" by Chatfield, C.

9. "Statistics.: Unlocking the Power of Data" by Robin H. Lock, Patti Frazer Lock, Kari Lock Morgan, and Eric F. Lock.

**Title of the Course:** Professional Ethics

**Course Code:** SOC-211

**Credit Hours:** 2+0

**Course Outline:**

**Understanding ethics in the helping professions:** Social Ethics, Business Ethics, Media Ethics, Police Ethics, Medical Ethics, Academia Ethics, Legal Ethics, and Research Ethics. professional activities, professional societies, professional competency and life-long learning. Theoretical issues in understanding ethics.

**Accountability and confidentiality issues:** Liability and Practice. Accountability and Auditing. Information/Position/Computer Misuse and the Criminal Laws. Intellectual property and cyber law. Resolving ethical dilemmas, Ethics and morals, The law and ethical practice, Ethical dimensions of E-professionalism,

**Professional responsibilities of practice:** Developing ethical literacy and personal care. Health and Safety at Work. Fulfilling Human Nature, Aristotle Nicomachean Ethics (selection), Kant

Fundamental Principles of the Metaphysics of Morals (Selection), Turning Values of Upside Down, Hearing the Feminine Voice.

**Suggested Instructional / Reading Materials:**

1. Applied Professional Ethics by Gregory R. Beabout, University Press of America (1993).
2. Ethics for the Information Age, Michael J. Quinn, 6th edition, Addison-Wesley (2015).
3. Human Relations: Interpersonal Job Oriented Skills, DuBrin, A. J. New York, Prentice Hall (2007).
4. Synder, C.R., & Lopez, S.J. (2007) Positive Psychology, USA, Sage Publications.
5. How to be a gentleman: A timely guide to timeless manners. Thomson Nelson

**Title of the Course**: Vector and Tensor Analysis

**Course Code:** MTH-221

**Credit Hours:** 3+0

**Course Outline:**

3-D vectors, summation convention, kronecker delta, Levi-Civita symbol, vectors as quantities transforming under rotations with ∈ijk notation, scalar- and vector-triple products, scalar- and vector-point functions, differentiation and integration of vectors, line integrals, path independence, surface integrals, volume integrals, gradient, divergence and curl with physical significance and applications, vector identities, Green’s theorem in a plane, divergence theorem, Stokes’ theorem, coördinate systems and their bases, the spherical-polar- and the cylindrical-coördinate meshes, tensors of first, second and higher orders, 41 algebra of tensors, contraction of tensor, quotient theorem, symmetric and skew-symmetric tensors, invariance property, application of tensors in modeling anisotropic systems, study of physical tensors (moment of inertia, index of refraction, etc.), diagnolization of inertia tensor as aligning coördinate frame with natural symmetries of the system

**Suggested Instructional / Reading Materials:**

1. Bourne DE, Kendall PC, Vector Analysis and Cartesian Tensors (2nd edition), Thomas Nelson Shah NA, Vector and Tensor Analysis, 2005, A-One Publishers, Lahore
2. Smith GD, Vector Analysis, Oxford University Press, Oxford
3. Spiegel MR, Vector Analysis, 1974, McGraw Hill, New York

**Title of the Course:** Entrepreneurship

**Course Code:** MGT-221

**Credit Hours:** 2+0

**Course Outline:**

**Introduction to Entrepreneurship:**

Definition and concept of entrepreneurship.

Why to become an entrepreneur?

Entrepreneurial process.

Role of entrepreneurship in economic development.

**Entrepreneurial Skills:**

Characteristics and qualities of successful entrepreneurs (including stories of successes and failures).

Areas of essential entrepreneurial skill and ability such as creative and critical thinking, innovation and risk taking abilities etc.

 **Opportunity Recognition and Idea Generation:**

Opportunity identification, evaluation and exploitation;

Innovative idea generation techniques for entrepreneurial ventures.

 **Marketing and Sales**

Target market identification and segmentation;

Four P's of Marketing.

Developing a marketing strategy.

Branding.

 **Financial Literacy:**

Basic concepts of income, savings and investments.

Basic concepts of assets, liabilities and equity.

Basic concepts of revenue and expenses.

Overview of cash-flows.

Overview of banking products including Islamic modes of financing.

Sources of funding for startups (angel financing, debt financing, equity financing etc.).

**Team Building for Startups:**

Characteristics and features of effective teams.

Team building and effective leadership for startups.

**Regulatory Requirements to Establish Enterprises in Pakistan:**

Types of enterprises (e.g., sole proprietorship; partnership; private limited companies etc.).

Intellectual property rights and protection.

Regulatory requirements to register an enterprise in Pakistan, with special emphasis on export firms.

Taxation and financial reporting obligation.

**Suggested Practical Activities (Optional)**

As part of the overall learning requirements, students shall be tasked with creating and presenting a comprehensive business plan at the end of the course for a hypothetical or real business idea. This practical exercise shall allow them to apply the knowledge, skills and competencies acquires in the course to develop a feasible business plan.

**Suggested Instructional / Reading Materials**

1. "Entrepreneurship: Successfully Launching New Ventures" by Bruce R. Barringer and R. Duane Ireland.
2. "Entrepreneurship: Theory, Process, and Practice" by Donald F. Kuratko.
3. "New Venture Creation: Entrepreneurship for the 21st Century" by Jeffry A. Timmons, Stephen Spinelli Jr., and Rob Adams.
4. "Entrepreneurship: A Real-World Approach" by Rhonda Abrams.
5. "The Lean Startup: How Today's Entrepreneurs Use Continuous Innovation to Create Radically Successful Businesses" by Eric Ries.
6. "Effectual Entrepreneurship" by Stuart Read, Saras Sarasvathy, Nick Dew, Robert Wiltbank,
7. and Anne-Valerie Ohlsson.

**Title of the Course:** Civics and Community Engagement

**Course Code:** SOC-221

**Credit Hours:** 03

**Course Outline:**

**Civics and Citizenship:**

Concepts of civics, citizenship, and civic engagement.

Foundations of modern society and citizenship.

Types of citizenship: active, participatory, digital, etc.

**State, Government and Civil Society:**

Structure and functions of government in Pakistan.

The relationship between democracy and civil society.

Right to vote and importance of political participation and representation.

**Rights and Responsibilities:**

Overview of fundamental rights and liberties of citizens under Constitution of Pakistan 1973.

Civic responsibilities and duties.

Ethical considerations in civic engagement (accountability, non-violence, peaceful dialogue, civility, etc.)

**Community Engagement:**

Concept, nature and characteristics of community. e Community development and social cohesion.

Approaches to effective community engagement.

Case studies of successful community driven initiatives.

**Advocacy and Activism:**

Public discourse and public opinion.

Role of advocacy in addressing social issues. e Social action movements.

**Digital Citizenship and Technology:**

The use of digital platforms for civic engagement.

Cyber ethics and responsible use of social media

Promoting inclusive citizenship and equal rights for societal harmony and peaceful co-existence.

**Diversity, Inclusion and Social Justice:**

understanding diversity in society (ethnic, cultural, economic, political etc.).

Youth. women and minorities engagement in social development.

Addressing social inequalities and injustice in Pakistan.

Promoting inclusive citizenship and equal rights for societal harmony and peaceful co-

existence

**Suggested Practical Activities (Optional)**

As part of the overall learning requirements, the course may have one or a combination of the following practical activities:

**Community Storytelling:** Students can collect and share stories from community members.

This could be done through oral histories, interviews, or multimedia presentations that capture

the lived experiences and perspectives of diverse individuals.

**Community Event Planning:** Students can organize a community event or workshop that

addresses a specific issue or fosters community interaction. This could be a health fair,

environmental cleanup, cultural festival, or educational workshop.

**Service-Learning:** Students can collaborate with a local nonprofit organization or community

group. They can actively contribute by volunteering their time and skills to address a particular

community need, such as tutoring, mentoring, or supporting vulnerable populations.

**Cultural Exchange Activities:** Students can organize a cultural exchange event that celebrate

the diversity within the community. This could include food tastings, performances, and presentations that promote cross-cultural understanding.

**Suggested Instructional / Reading Materials**

1. "Civics Today: Citizenship, Economics, & You" by McGraw-Hill Education.
2. "Citizenship in Diverse Societies" by Will Kymlicka and Wayne Norman.
3. "Engaging Youth in Civic Life" by James Youniss and Peter Levine.
4. "Digital Citizenship in Action: Empowering Students to Engage in Online Communities" by Kristen Mattson.
5. "Globalization and Citizenship: In the Pursuit of a Cosmopolitan Education" by Graham Pike and David Selby.
6. "Community Engagement: Principles, Strategies, and Practices" by Becky J. Feldpausch and
7. Susan M. Omilian. "Creating Social Change: A Blueprint for a Better World" by Matthew Clarke and Marie- Monique Steckel.

**Title of the Course:** Differential Equations

**Course Code:** MTH-222

**Credit Hours:** 3+0

**Course Outline:**

Review of ordinary differential equations with constant and variable coefficients, Methods of undetermined coefficients and variation of parameters, Existence and uniqueness of solutions, Systems of linear ordinary differential equations, Modeling of physical systems and their solutions, Simple harmonic motion, Damped vibrations, Forced vibrations. Series solutions of differential equations, Power series method,

Method of Frobenius, Bessel, Legendre, Lagguere, Hermite, and Hypergeometric differential equations and their solutions, Sturm–Liouville systems, Green’s function for ordinary differential equations, Introduction of partial differential equations.

**Suggested Instructional / Reading Materials:**

1. Shah, N.A. (2011) “Ordinary Differential Equations” A–One Publishers, Urdu Bazar, Lahore.
2. Rainville, E.D. and Bedient, P.E. (1989) “Elementary Differential Equations” (Seventh Edition) Macmillan Publishing Company.
3. Brikhoff, G. and Rota G. C. (1989) “Ordinary Differential Equations” (Fourth Edition) John–Wiley & Sons Inc.
4. Spiegel, R. (1981) “Applied Differential Equations” Prentice Hall.
5. Bronson R. (1994) “Theory and Problems of Differential Equations” (Second Edition)
6. Schaum’s Outline Series, McGraw–Hill.
7. Kreyszig, E. (1999) “Advanced Engineering Mathematics (Eighth Edition)” John–Wiley & Sons Inc.

**Title of the Course**: Quantitative Reasoning-II

**Course Code:** QR-221

**Credit Hours**: 3+0

**Course Outline:**

**Logic, Logical and Critical Reasoning**

Introduction and importance of logic;

Inductive, deductive and abductive approaches of reasoning;

Propositions, arguments (valid; invalid), logical connectives, truth tables and propositional equivalences;

Logical fallacies;

Venn Diagrams;

Predicates and quantifiers;

Quantitative reasoning exercises using logical reasoning concepts and techniques.

**Mathematical Modeling and Analyses**

Introduction to deterministic models;

Use of linear functions for modeling in real-world situations;

Modeling with the system of linear equations and their solutions;

Elementary introduction to derivatives in mathematical modeling;

Linear and exponential growth and decay models;

Quantitative reasoning exercises using mathematical modeling.

**Statistical Modeling and Analyses**

Introduction to probabilistic models;

Bivariate analysis, scatter plots;

Simple linear regression model and correlation analysis;

Basics of estimation and confidence interval;

Testing of hypothesis (z-test; t-test);

Statistical inference in decision making;

Quantitative reasoning exercises using statistical modeling.

**Suggested Instructional / Reading Materials**

1. "Using and Understanding Mathematics: A Quantitative Reasoning Approach" by Bennett, J. 0., Briggs, W. L., & Badalamenti, A.
2. "Discrete Mathematics and its Applications" by Kenneth H. Rosen. 3. "Discrete Mathematics with Applications" by Susanna S. Epp.
3. "Applied Mathematics for Business, Economics and Social Sciences" by Frank S Budnick. 5. "Elementary Statistics: A Step by Step Approach" by Allan Bluman.
4. "Introductory Statistics" by Prem S. Mann.
5. "Applied Statistical Modeling" by Salvatore Babones.
6. "Barrons SAT" by Sharvon Weiner Green, M.A and Ira K.Wolf.

**Title of the Course:**  Social Psychology

**Course Code:** SOC-222

**Credit Hours**: 2+0

**Course Outline:**

**Introduction to social psychology:** What is social psychology, Social psychology and human values, Is social psychology simply common sense?

**Self- Presentation and Social Perception:** Self concept – Who am I?, Nonverbal behavior, Attribution, Impression Management.

**Social Cognition**: Schemas, Heuristics, Affect and Cognition.

**Behavior and Attitudes**: Nature of attitudes, Formation, maintenance, and change in attitudes, Relationship between attitude and behavior, Cognition & attitude.

**Aspects of Social Identity:** The self - Nature of the self , Self-concept , Social diversity, Self-esteem Other aspects of self functioning - Self focusing, Cognitive and affective aspects, Self Monitoring, Self Efficacy. Gender & socialization

**Social Influence:** Conformity, Compliance, Obedience

**Pro-social Behavior:** Why do we help? When do we help? Does true altruism really exist? Whom do we help? How we can increase helping?

**Aggression/hurting others:** What is Aggression? Influences on Aggression. Reducing Aggression. Lucifer effect. Bullying.

**Conflict and peacemaking:** What creates conflicts? Social Dilemmas, Competition, Perceived Injustice, Misperception. How Can Peace Be Achieved? Contact, Communication, Conciliation.

**Social Psychology and the Sustainable Future:** An environmental call to action, Enabling sustainable living, The social psychology of materialism and wealth.

**Suggested Instructional / Reading Materials:**

1. Wesley. Fisher, R. J. (1982). Social psychology: An applied approach. New York: St.MartinPress.
2. Forsyth,D.F.(1987). Social Psychology.California: BrooksPublishing Company.
3. Myers, D. G. (1987).Exploring social psychology. New York: McGraw-Hill.
4. Myers, D. G. (1987).Social Psychology. New York: McGraw-Hill.
5. Wayant, J. M. (1986). Applied social psychology. New York: OxfordUniversityPress.

**Title of the Course**: Real Analysis-I

**Course Code:** MTH-311

**Credit Hours:** 3+0

**Course Outline**:

Real number system, Ordered sets, Bounded sets, Real field and the extended real number system, Euclidean spaces, Equivalent sets, Countable and uncountable sets, Concept of cardinality numbers, Addition and multiplication of cardinals, Neighbourhood of a point, Limit point, Isolated point, Open , closed, perfect, dense, compact and connected sets. Numerical sequences and series, Convergent and divergent sequences and series, Subsequences, Cauchy sequences. Completeness, Infinite series, Tests of convergence, Power series, Continuous functions, Properties of continuous functions on closed and bounded sets, Discontinuities, Uniform continuity, Differentiability, Mean value theorem and L’ Hospital rule, Error estimates.

**Suggested Instructional / Reading Materials**

1. Rudin, W. (1976) “Principles of Mathematical Analysis” McGraw–Hill.
2. Kaplan, W. (1959) “Advanced Calculus” Addison Wesley.
3. Apostol, T. M. (1985) “Mathematical Analysis” Addison Wesley.
4. Douglass, S. A. (1996) “Introduction to Mathematical Analysis” Addison Wesley.
5. Malik, S. C. , Savita. A. (1991) “Mathematical analysis” Wiley Eastern Ltd.

**Title of the Course**: Numerical Methods with Programming

**Course Code:** MTH-312

**Credit Hours:** 3+1

**Course Outline**:

**Number Systems and Errors:**

Round off errors and computer arithmetic, Error estimation, Floating point arithmetic.

**Solution of Non-Linear Equations:**

Iterative methods and convergence: Bisection method, fixed point iterative method, Regula Falsi, Secant and Newton’s method.

**System of Linear Equations**

Direct methods: Gaussian elimination method, Gauss-Jordan method, matrix inversion method, factorization (Doolittle, Crout and Cholesky) method and its various forms, Iterative methods, and convergence: Gauss-Jacobi method and Gauss-Seidel method, Ill-condition system and condition number, Eigen values and eigenvectors, Power and Rayleigh quotient method.

**Interpolation and Polynomial Approximation**

Difference operators, Interpolation with unequal intervals: Lagrange’s interpolation formula, Newton’s divided difference formula, Error in polynomial interpolation, Interpolation with equal intervals: Gregory Newton forward/backward interpolation formula, Error in polynomial interpolation, Central difference interpolation formulae: Gauss’s forward/backward interpolation formula, Stirling’s formula, Laplace Everett’s formula, Bessel’s formula.

**Suggested Instructional / Reading Materials**

1. Burden R. L. and Fairs J. D. (1997) “Numerical Analysis”, (Sixth edition) Brooks/Cole Publishing Co.
2. Hoffman D. J. (1992) “Numerical Methods for Engineers and Scientists” McGraw-Hill.
3. Gerald C.F. (1998) “Applied Numerical Analysis”, (sixth edition), Addison-Wesley
4. Mathews J. H. (1992) “ Numerical Methods for Mathematics, Science and Engineering”, (second edition) Prentice Hall.
5. Faiz Ahmad and M. Afzal Rana (1995) “Numerical Analysis”, National Book Foundation.

**Title of the Course**: Group Theory

**Course Code:** MTH-313
**Credit Hours:** 3+0

**Course Outline:** Definition of a group, subgroup, subgroup generated by a set. The cyclic groups, cosets and Lagrange’s theorem. Normalizer centralizer. The center of a group. Equivalence relation in a group, conjugacy classes. Normal subgroups, quotient group. Group homomorphisms: Homomorphisms and isomorphism and Automorphism. Kernel and image of homomorphism. Isomorphism theorems. Permutation groups. The cyclic decomposition of a permutation group. Cayley’s theorem. Direct product of two groups and examples.

**Suggested Instructional / Reading Materials:**

1. J. B. Fraleigh, A First Course in Abstract Algebra, Addison-Wesley Publishing Company, 2002.
2. J. S. Rose, A Course on Group Theory, Cambridge University Press, 1978.
3. I. N. Herstein, Topics in Algebra, Xerox Publishing Company, 1964.
4. P. M. Cohn, Algebra, John Wiley and Sons, London, 1974.

**Title of the Course**: Complex Analysis

**Course Code:** MTH-314

**Credit Hours**: 3+0

**Course Outline:**

**Functions of Complex Variables:** Definition, limit and continuity, Branches of functions, Differentiable and analytic functions. The Cauchy-Riemann equations, Entire functions, Harmonic functions, Elementary functions: The exponential, Trigonometric, Hyperbolic, Logarithmic and Inverse elementary functions, Open mapping theorem. Maximum modulus theorem.

**Complex Integrals:** Contours and contour integrals, Cauchy-Goursat theorem, Cauchy integral formula, Lioville’s theorem, Morerea’s theorem.

**Series:** Power series, Radius of convergence and analyticity, Taylor’s and Laurent’s series, Integration and differentiation of power series.

**Singularities, Poles and residues**: Zero, singularities, Poles and Residues, Types of singular points, Calculus of residues, contour integration, Cauchy’s residue theorem with applications. Mobius. transforms, Conformal mappings and transformations

**Suggested Instructional / Reading Materials:**

1. R. V. Churchill, J. W. Brown, Complex Variables and Applications , 5 th edition, McGraw Hill, New York, 1989.
2. J. H. Mathews and R. W. Howell, Complex Analysis for Mathematics and Engineering, 2006.
3. S. Lang, Complex Analysis, Springer-Verlag, 1999.
4. R. Remmert, Theory of Complex Functions, Springer-Verlag, 1991.
5. W. Rudin, Real and Complex Analysis, McGraw-Hill, 1987.

**Title of the Course**: Metrics and Topology

**Course Code:** MTH-315

**Credit Hours:** 3+0

**Course Outline**:

**Metric Spaces**: Definition and various examples of metric spaces, Holder’s inequality, Cauchy-schwarz and minkowski’s inequality, Open and closed balls, Neighborhoods, Open and closed sets, Interior, Exterior and boundary points, Limit points, Closure of a set, Convergence in metric spaces, Cauchy sequences, Continuity in metric spaces, Inner product and norm, Orthonormal sets and basis, The Gram-Schmidt process

**Topology**: Examples; open and closed subsets, metric spaces, neighbourhoods. Examples. Limit points and accumulation points. Interior, closure, dense subsets. Constructing new topological spaces: Cartesian products, induced topology and quotient topology. Continuous maps, open and closed maps, homeomorphisms. Examples: R, RxR, S^1, S^2, torus, cylinder. Cauchy sequences, complete metric spaces. Separation axioms. Compact spaces. Properties. Power of Compactness. Image of a compact set through a continuous map. Compactness and completeness of metric spaces. Connected spaces, connected components. Properties. Image of a connected set through a continuous map. Path-connectedness.

**Suggested Instructional / Reading Materials:**

1. Micheal, O. Searcoid, Metric Spaces, Springer, 2007
2. E. Kreyszig, Introduction to Functional Analysis with Applications, John Wiley and Sons, 1978
3. W.A. Sutherland, Introduction to Metric and Topological Spaces, Clarendon Press Oxford, 1975
4. J. Dugundji, Topology, (Allyn and Bacon Inc., Boston 1966)
5. G. F. Simmon, Introduction to Topology and Modern Analysis, (McGraw Hill Book Company, New York, 1963)
6. Stephen Willard, General Topology, (Addison-Wesley Publishing Co., London, 1970)
7. Seymour Lipschutz, General Topology, (Schaum's Outline Series, McGraw Hill Book Company 2004) 5. James R. Munkres, Topology, 2nd edition, (Prentice Hall Inc., 2003)

**Title of the Course**: Mathematical Methods of Physics

**Course Code:** MTH-321

**Credit Hours:** 3+0

**Course Outline**:

**Special Functions:** Bessel Functions; Neumann Functions; Henkel Functions; Spherical Bessel Functions; Legendre Functions; Associated Legendre Functions; Spherical Harmonics; Hermite Polynomials.

**Partial Differential Equations (PDEs):** Introduction to Important PDEs in Physics (Wave Equation; Diffusion Equation; Poisson’s Equation; Schrodinger’s Equation); General form of Solution; General and Particular Solutions (First Order; Inhomogeneous; Second Order); Characteristics and Existence of Solutions; Uniqueness of Solutions; Separation of Variables in Cartesian Coordinates; Superposition of Separated Solutions; Separation of Variables in Curvilinear Coordinates; Special Functions; Integral Transform Methods; Green’s Functions.

**Complex Analysis:** Review (Polar form of Complex Numbers and De Moivre’s Theorem; Complex Logarithms and Powers); Functions of a Complex Variable; Cauchy-Riemann Conditions; Power Series in a Complex Variable and Analytic Continuation with Examples; Multi-valued Functions and Branch Cuts; Singularities and Zeroes of Complex Functions; Complex Integration; Cauchy’s Theorem; Cauchy’s Integral Formula; Laurent Series and Residues; Residue Integration Theorem; Definite Integrals using Contour Integration.

**Suggested Instructional / Reading Materials:**

1. G. Arfken, H. J. Weber, and F. E. Harris, “Mathematical Methods for Physicists”, Academic Press, 7th ed. (2012).
2. K. F. Riley, M. P. Hobson, S. J. Bence, “Mathematical Methods for Physicists”, Cambridge University Press, (2006).
3. E. Kreyszig, “Advanced Engineering Mathematics”, John Wiley, 8th ed. (1999).

**Title of the Course**: Differential Geometry

**Course Code:** MTH-322

**Credit Hours:** 3+0

**Course Outline**:

Space curves, Moving trihedron, Curvature, Torsion, Serret–Frenet formulae, Osculating plane, circle and sphere, Helices, Indicatrices and their curvature and torsion, Fundamental theorem of space curve, Involutes, Evolutes. The theory of surfaces, Tangent and normal planes, Envelopes and characteristics related to one parameter family of surfaces, Edge of regression, Developable surfaces and developable associated with a space curve, Parametric curves, Two fundamental forms of curves on a surface, Meunier’s theorem, Lines of curvatures, Principal directions and principal curvature, Euler’s theorem, Geodesics and Geodesic equations.

**Suggested Instructional / Reading Materials**

1. Weatherburn, C. E. (1964) Vol. 1 “Differential Geometry” Cambridge University Press.
2. Stoker, J. J. (1969) “Differential Geometry”, John Wiley and Sons.
3. Goetz, A. (1970) “Introduction to Differential Geometry, Addison Wesley.
4. Kreyszig, E. (1991) “Differential Geometry” Dover Publications.
5. Lipschutz, M., et. al. (1983) “Differentiable Geometry” Schaum’s Outline Series, McGraw–Hill.

**Title of the Course**: Functional Analysis-I

**Course Code:** MTH-323

**Credit Hours:** 3+0

**Course Outline**:

Holder and Minkowski’s Inequalities, Review of Metric Spaces with emphasis on convergence and completeness, Contraction mapping and Banach fixed point theorem, Separable spaces, Normed spaces, Quotient spaces, Equivalent norms, Banach spaces, Compactness, Bounded linear mappings and Bounded linear functionals, Linear operators and functionals on Finite dimensional spaces, Dual spaces, Inner product spaces, Parallelogram law, Schwarz inequality, Hilbert spaces, Orthogonality, Orthonormal sequences, Gram–schmidt process, Representation of functionals on Hilbert spaces. Applications of Hahn-Banach Theorems.

**Suggested Instructional / Reading Materials**

1. Kreyszig, E. (1989) “Introductory Functional Analysis with Applications” John Wiley and Sons.
2. Taylor, A. E. (1958) “Introduction to Functional Analysis”. John Wiley and Sons.
3. Rudin, E. (1973) “Functional Analysis” McGraw–Hill.
4. Riesz,F. and Nagy, S. G. (1965) “Functional Analysis”. Fredric Ungar Publishing Co. U.S.A.
5. Krishnan, V. K. (2001) “Text Book of Functional Analysis” Prentice Hall.

**Title of the Course**: Discrete Mathematics

**Course Code:** MTH-324

**Credit Hours:** 3+0

**Course Outline**:

Introduction to Logic, Propositional calculus, Sets, Sequences and Functions, Growth of functions. Algorithms, Complexity of algorithms, Integers and Division, Applications of Number theory, Matrices. Mathematical Reasoning: Methods of Proof, Mathematical Induction, Recursive definitions. Advanced Counting techniques: Recurrence Relations, Solving Recurrence Relations, Generating Functions, Inclusion-Exclusion, Applications of Inclusion-Exclusion. Relations: Relations and their properties, Representing Relations, Closure of Relations, Equivalence Relations, Partial Orderings. Graphs: Introduction to Graphs, Graph Terminology, Representing Graphs and Graph Isomorphism, Connectivity, Euler and Hamilton Paths, Shortest Path problems, Planner Graphs, Graph Coloring. Trees: Introduction to Trees, Spanning Trees, Minimum Spanning Trees.

**Suggested Instructional / Reading Materials**

1. Rosen K. H (1999) “Discrete Mathematics and its Applications”, (Fourth edition) McGraw-Hill.
2. Susana S. Epp. (1999) “Discrete Mathematics with Applications”, (Second Edition) PWS Publishing Company.
3. Ross K.A and Wright C.R.B (1992) “Discrete Mathematics” (Third edition) Prentice Hall.
4. Dymacek W. M. and Sharp H. Jr. (1998) “Introduction to Discrete Mathematics” Web McGraw-Hill.
5. Goodaire E. G. and Parameter M.M. (1998) “ Discrete Mathematics with Graph Theory”, Prentice Hall, Inc.

**Title of the Course**: Operations Research-I

**Course Code:** MTH-325

**Credit Hours:** 3+0

**Course Outline**:

Introduction and Impact of Operations Research, Techniques and Modeling of Operations Research problems, Construction of Linear Programming model, Graphical solution and graphical sensitivity analysis, Transition from graphical to algebraic solution, Simplex method, Special cases in Simplex method applications, Duality and Sensitivity Analysis, Primal-Dual relationships, Economics interpretation of duality, Dual Simplex method, Generalized Simplex algorithm, Computer solution and Sensitivity analysis of LP problems.

Transportation problem: Types of transportation problem, Methods to solve transportation problem, Non-traditional transportation models, Transportation algorithm, Assignment Model, Transshipment model, Computer solution of transportation models.

**Suggested Instructional / Reading Materials**

1. HAMDY A. TAHA. (2002)“Operations Research: An introduction” (Seventh Edition), Prentice – Hall of India.·
2. FREDERICK S. HILLIER, G.J. LIEBERMON, (2001) “Introduction to Operations Research” (seventh Edition ) Mc-Graw Hill.
3. R. RANNEERSELVAM, (2002) “Operations Research” Prentice Hall of India.
4. Harvey M. Wagner (2001) “Principles of Operations Research ”, Prentice Hall India.
5. Richard B. (2001) “Operations Research”, Schaum’s Outline Series ,McGraw Hill

**Title of the Course**: Abstract Algebra-I

**Course Code:** MTH-411

**Credit Hours:** 3+0

**Course Outline**:

**Basic Notions**

Sets, functions; binary operations; modular arithmetic.

**Rings**

Mathematical theory of a rings, subrings, integral domains, fields, and division ring; interconnections between these algebraic structures; commutative and noncommutative rings; zero divisors; characteristic and other fundamental ring theoretic topics. Polynomials

Polynomials; the division algorithm; factorization; units; associates; unique factorization domains. **Ideals and Quotient Rings**

 Ring homomorphisms; ideals; quotient rings; homomorphism theorem for rings; quotients of polynomial rings.

**Groups Definition;** elementary properties including cancellation laws, uniqueness of the identity and inverses; unique solvability of linear equations; subgroups and subgroup tests; orders of elements; cyclic groups; modular systems; abelian groups; permutation groups, including the alternating and symmetric groups, cycle notation, and transpositions; dihedral groups and applications to symmetry.

**Suggested Instructional / Reading Materials**

1. Abstract Algebra: An Introduction, 3rd Edition by Thomas W. Hungerford

**Title of the Course**: Introduction to Mathematical Modelling

**Course Code:** MTH-412

**Credit Hours:** 3+0

**Course Outline**:

**Functions; modeling with linear**

a. Function definition; domain and range b. Functions described by tables, graphs and formulas c. Increasing and decreasing functions; local and absolute extrema d. Concavity; inflection points e. Average rate of change f. Linear functions with applications g. Slope-intercept and point-slope forms h. Piecewise-linear functions with applications

**Linear regression; modeling with exponential functions**

a. Fitting linear models to data b. Evaluating model error; the sum of squared errors c. Interpreting the correlation coefficient d. Exponential growth functions with applications e. Growth factors and rates f. Doubling time g. Compound interest h. Exponential decay functions with applications i. Decay factors and rates j. Half-life

**Additional topics in exponential modeling, modeling with logarithmic functions; linear systems**

a. Fitting exponential models to data b. Continuous compounding c. Continuous growth and decay d. Newton’s law of cooling and heating e. Logarithmic functions with applications f. Fitting logarithmic models to data g. Matrices h. Representing a system of linear equations with a matrix equation i. Solving linear systems via matrix equations

**Modeling with polynomial functions**

a. Quadratic functions with applications b. Projectile motion c. Maxima and minima applications d. Fitting quadratic models to data e. Interpreting the coefficient of determination f. Polynomial functions of higher degree with applications g. Polynomial interpolation h. Fitting cubic and quartic models to data.

**Suggested Instructional / Reading Materials**

1. Functions, Data, and Models, Gordon and Gordon, The Mathematical Association of America, 2010 (ISBN-10: 0-8838-5767-7; ISBN-13 978-0-88385-767-0).
2. An introduction to mathematical modeling by Bender, Edward A. Courier Corporation, 2000

**Title of the Course:** Number Theory

**Course Code:** MTH-421

**Credit Hours:** 3+0

**Course Outline:**

 Preliminaries: Well-ordering principle. Principle of finite induction. Divisibility theory: The division algorithms. Basis representation theorem. Prime and composite numbers. Canonical decomposition. The greatest common divisor. The Euclidean algorithm. The fundamental theorem of arithmetic. Least common multiple. Linear Diophantine equations: Congruences. Linear congruences. System of linear congruences. The Chinese remainder theorem. Divisibility tests. Solving polynomial congruences. Fermat's and Euler's theorems. Wilson's theorem. Arithmetic functions: Euler's phi-function. The functions of J and sigma. The Mobius function. The sieve of Eratosthenes. Perfect numbers. Fermat and Mersenne primes. Primitive Roots and Indices: The order of an integer mod n. Primitive roots for primes. Composite numbers having primitive roots. Quadratic residues: Legendre symbols and its properties. The quadratic reciprocity law. Quadratic congruences with composite moduli. Pythagorean triples. Representing numbers as sum of two squares.

**Suggested Instructional / Reading Materials:**

1. D.M. Burton, Elementary Number Theory, McGraw-Hill, 2007.

2. W.J. Leveque, Topics in Number Theory, vols. I and II, Addison- Wesley, 1956.

3. S.B. Malik , Basic Number Theory, Vikas Publishing house, 1995.

4. K.H. Rosen, Elementary Number Theory and its Applications, 5th edition, Addison-Wesley, 2005.

5. I. Niven, H.S. Zuckerman, H.L. Montgomery, An Introduction to the theory of Numbers, John Wiley and Sons, 1991.

6. A. Adler, J.E. Coury, The Theory of Numbers, Jones and Bartlett Publishers, 1995.

**Title of the Course**: Numerical Linear Algebra

**Course Code:** MTH-422

**Credit Hours:** 3+0

**Course Outline**:

**Fundamentals**

Matrix-vector multiplication b. Orthogonal vectors and matrices c. Norms d. Computer arithmetic

**Singular Value Decomposition**

**QR Factorization and Least Squares**

Projectors b. QR factorization c. Gram-Schmidt orthogonalization d. Householder triangularization e. Least squares problems

**Conditioning and Stability**

Conditioning and condition numbers b. Stability

**Systems of Equations**

Gaussian elimination b. Cholesky factorization

**Eigenvalues**

Overview of eigenvalue algorithms b. Reduction to Hessenberg or tridiagonal form c. Rayleigh quotient, inverse iteration d. QR Algorithm without and with shifts e. Computing the SVD

**Iterative Methods**

**Suggested Instructional / Reading Materials**

1. Lloyd N. Trefethen and D. Bau, Numerical Linear Algebra, SIAM (1997), ISBN 0-89871-361-7.
2. D. Kincaid and W. Cheney, Numerical Analysis: Mathematics of Scientific Computing, 3rd Ed, Brooks/Cole (2002), ISBN 0-534-38905-8.

**Title of the Course:** Real Analysis-II

**Course Code:** MTH-401

**Credit Hours:** 3+0

**Course Outline:**

**The Riemann-Stieltjes Integrals:** Definition and existence of integrals, Properties of integrals, Fundamental theorem of calculus and its applications, Change of variable theorem, Integration by parts.

**Functions of Bounded Variation**: Definition and examples, Properties of functions of bounded variation

**Improper Integrals**: Types of improper integrals, Tests for convergence of improper integrals, Beta and gamma functions, Absolute and conditional convergence of improper integrals

**Sequences and Series of Functions**: Power series, Definition of point-wise and uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, Examples of uniform convergence

**Suggested Instructional / Reading Materials**:

 1. W. Rudin, Principles of Mathematical Analysis, 3rd edition, (McGraw Hill 1976)

2. R. G. Bartle, Intoduction to Real analysis, 3rd edition, (John Wiley and sons, 2000)

3. T. M. Apostol, Mathematical Analysis, (Addison-Wesley Publishing Co., 1974)

4. A. J. Kosmala, Introductory Mathematical Analysis, (WCB company, 1995)

5. W. R. Parzynski and P. W. Zipse, Introduction to Mathematical Analysis, (Mc Graw Hill company, 1982)

6. H. S. Gaskill and P. P. Narayanaswami, Elements of Real Analysis, (Printice Hall, 1988)

**Title of the Course**: Advanced Group Theory

**Course Code:** MTH-402

**Credit Hours:** 03

**Course Outline:**

Introduction to Sets and Structures. Examples of groups. Finite groups. Subgroups. Permutations and cyclic groups. Isomorphism’s and Homomorphism with separate reference to Abelian groups. Cosets, Normal groups, Factor groups and Simple groups. Classification of Finite Simple Groups. Series in groups. Zassenhaus lemma, Normal series and their refinements Composition series.  The Jordan Holder Theorem. The Sylow theorems. Linear groups, types of linear groups. Representation of linear groups. The projective special linear groups. Free groups and group presentations.

**Suggested Instructional / Reading Materials:**

1. J. B. Fraleigh, A First Course in Abstract Algebra, Addison-Wesley Publishing Company, 2002.
2. J. S. Rose, A Course on Group Theory, Cambridge University Press, 1978.
3. I. N. Herstein, Topics in Algebra, Xerox Publishing Company, 1964.
4. P. M. Cohn, Algebra, John Wiley and Sons, London, 1974.

**Title of the Course:** Fuzzy Set Theory

**Course Code:** MTH-403

**Credit Hours:** 3+0

**Course Outline:**

Introduction to Fuzzy sets , Crisp vs Fuzzy Types of Fuzzy sets, Membership functions , Alpha cuts Contd alpha cuts, Operation on fuzzy sets, t-norm, complements t-conorm, combination of operartions continued, Introduction to Fuzzy arithmetic Interval arithmetic, +,-,,\* using alpha cuts MIN and MAX fuzzy numbers, Fuzzy arithmetic using Alpha cuts continued Decomposition principle, Extension principle Fuzzy arithmetic using Extension Principle Fuzzy Equations, Relations, Introduction to fuzzy relations Projections, Equivalence relation, transitive closure, compatibility relation, Introduction to propositional Logic, Boolean Algebra Multi valued logic , Fuzzy Logic, Linguistic hedges, Fuzzy propositions (conditional and unconditional), Inference from conditional and qualified fuzzy propositions, Fuzzy Quantifiers, Inference from quantified fuzzy propositions, Introduction to possibility theory Possibility vs probability Belief and Plausibility, Dempsters rule

**Suggested Instructional / Reading Materials:**

1. Fuzzy Sets and Fuzzy Logic, Geoge J. Klir and Bo Yuan
2. Fuzzy Mathematical Models in Engineering and Management Science, A. Kaufmann and M.M. Gupta
3. Fuzzy Logic, Timothy J. Ross
4. Fuzzy Set Theory, H.J. Zimmermann
5. Introduction to Fuzzy Logic and Fuzzy Sets, J.J. Buckley and E. Eslami

**Title of the Course**: Introduction to Graph Theory

**Course Code:** MTH-404

**Credit Hours:** 3+0

**Course Outline**:

Definition and examples of a graph, Sub graph, Types of graphs, Paths, cycles, wheels and walks, Connected and disconnected graph, Isomorphism, Handshaking lemma, Matrix representation of a graph, Three puzzles, Connectivity, Eulerian graphs, Hamiltonian graphs, Shortest path Algorithms, Trees, Traversability, Matchings and Factorizations, Colouring of Graphs

**Suggested Instructional / Reading Materials:**

1. Robin J. Wilson, Graph Theory, 4th Edition, Longman 2000
2. Douglas B. West, Graph Theory, 2nd Edition, Prentice Hall 2003
3. V.K.Balakrishnan, Graph Theory, Schaum’s Ouline 1997
4. Parthasarathy, K.R. Basic Graph Theory, McGraw Hill, 1994
5. Bela Bollobas, Graph Theory, Springer Verlag, New York, 1979

**Title of the Course**: Measure Theory and Lebesgue Integration

**Course Code:** MTH-405

**Credit Hours:** 3+0

**Course Outline**:

**Measurable Sets**: Outer measure, Lebesgue measure, Lebesgue measurable sets, Borel sets, Non measurable sets

**Measurable** **Functions:** Lebesgue measurable functions, Simple functions, characteristic functions, Borel measurable function, Littlewood three principle

**The Lebsegue Integration**: Review of the Riemann integral, Lebsegue integral, Integral of a non negative function, Integral of measurable functions, Convergence in measure

**Suggested Instructional / Reading Materials**:

1. D. Smith, M. Eggen and R. St. Andre, A Transition to Advanced Mathematics, (Brooks, 2001)
2. 2 Seymour Lipshcutz, Set Theory and Related Topics, (Mc-Graw Hill Book Company, 1999) 3 H. L. Royden, Real Analysis, (Macmillam, 1968)
3. D. L. Cohan, Measure Theory, (Bir Khauser, 1980)
4. P.R. Halmos, Measure Theory, (Von Nostrand, New York, 1950)

**Title of the Course**: Fixed Point Theory and Applications

**Course Code:** MTH-406

**Credit Hours:** 3+0

**Course Outline**:

Review of fixed points; Fixed point iteration procedure; Fixed point formulation typical functional equations; Picard iteration, Banach’s fixed point theorem, Theorem of Edelstein, Kannan’s fixed point theorem, Chatterjee’s fixed point theorem, Reich’s fixed point theorem, Hardy Roger’s fixed point theorem, Ciric’s fixed point theorem, Fixed point theorems on closed ball, fixed point theorems in ordered Metric Spaces, Generalized contractions, Admissible mapping, dominated mapping, Weakly compatible mappings, commutable mappings. Fixed point results for two, three and four mappings.

**Suggested Instructional / Reading Materials**:

1. [Topics in Metric Fixed Point Theory by Kazimierz Goebel; W. A. Kirk](http://catalog.riphah.edu.pk:801/cgi-bin/koha/opac-detail.pl?biblionumber=13619), Call Number: 515.7248 GOE, ISBN: 9780521064064, Publication Date: 2008-06-05
2. [An Introduction to Metric Spaces and Fixed Point Theory by Mohamed A. Khamsi; William A. Kirk](http://catalog.riphah.edu.pk:801/cgi-bin/koha/opac-detail.pl?biblionumber=13401),

**Title of the Course**: Functional Analysis-II

**Course Code:** MTH-407

**Credit Hours:** 3+0

**Course Outline**:

**Compact Normed Spaces:** Completion of metric spaces, Completion of normed spaces, Compactification, Nowhere and everywhere dense sets and category, Generated subspaces and closed subspaces, Factor Spaces, Completeness in the factor spaces

**Complete Orthonormal Set:** Complete orthonormal sets, Total orthonormal sets, Parseval’s identity, Bessel’s inequality

**The Specific geometry of Hilbert Spaces:** Hilbert spaces, Bases of Hilbert spaces, Cardinality of Hilbert spaces, Linear manifolds and subspaces, Othogonal subspaces of Hilbert spaces, Polynomial bases in L2 Spaces.

**Suggested Instructional / Reading Materials**

* + - 1. G. Bachman and L. Narici, Functional Analysis, (Academic Press, New York, 1966)
			2. A. E. Taylor, Functional Analysis, (John Wiley and Sons, Toppan, 1958)
			3. G. Helmberg, Introduction to Spectral theory in Hilbert spaces, (North Holland Publishing Company, 1969)
1. E. Kreyszig, Introduction to Functional Analysis with Applications, (John Wiley and Sons, 2004)
2. F. Riesz and B. Sz. Nagay, Functional Analysis, (Dover Publications, Inc., New York, Ungar, 1965)

**Title of the Course**: Combinatorial Mathematics

**Course Code:** MTH-408

**Credit Hours:** 3+0

**Course Outline**:

The course covers the basic combinatorial techniques as well as introduction to more advanced ones. The topics discussed include elementary counting, the pigeonhole principle, counting spanning trees, Inclusion-Exclusion, generating functions, Ramsey Theory, Extremal Combinatorics, Linear Algebra in Combinatorics, introduction to the probabilistic method, spectral graph theory, topological methods in combinatorics.

**Suggested Instructional / Reading Materials**

1. [Applied combinatorics by Tucker, Alan](http://catalog.riphah.edu.pk:801/cgi-bin/koha/opac-detail.pl?biblionumber=13395), Call Number: 511.6 TUC, ISBN: 9788126526420, Publication Date: 2007.
2. [Combinatorics : An Introduction (1)](http://site.ebrary.com/lib/hec/detail.action?adv.x=1&docID=10915819&f00=all&f01=subject&p00=combinatorics&p01=%22Combinatorial+Analysis%22) by Faticoni, Theodore G.
Date Published: 2014.
3. [Analytic Combinatorics](http://site.ebrary.com/lib/hec/detail.action?adv.x=1&docID=10277515&f00=all&f01=subject&p00=combinatorics&p01=%22Combinatorial+Analysis%22), by Flajolet, Philippe, Sedgewick, Robert,Sedgewick, Robert
4. [Wiley Series in Discrete Mathematics and Optimization : Introduction to Combinatorics (2)](http://site.ebrary.com/lib/hec/detail.action?adv.x=1&docID=10895758&f00=all&f01=subject&p00=combinatorics&p01=%22Combinatorial+Analysis%22), by Erickson, Martin J., DATE PUBLISHED June 2013

**Title of the Course**: Theory of Partial Differential Equations

**Course Code:** MTH-409

**Credit Hours:** 3+0

**Course Outline**:

**First order PDEs:**Introduction, formation of PDEs, solutions of PDEs of first order, The Cauchy’s problem for quasilinear first order PDEs, First order nonlinear equations, Special types of first order equations **Second order PDEs:**Basic concepts and definitions, Mathematical problems, Linear operators, Superposition, Mathematical models: The classical equations, the vibrating string, the vibrating membrane, conduction of heat solids, canonical forms and variable, PDEs of second order in two independent variables with constant and variable coefficients, Cauchy’s problem for second order PDEs in two independent variables

**Methods of separation of variables:**Solutions of elliptic, parabolic and hyperbolic PDEs in Cartesian and cylindrical coordinates **Laplace transform:**Introduction and properties of Laplace transform, transforms of elementary functions, periodic functions, error function and Dirac delta function, inverse Laplace transform, convolution theorem, solution of PDEs by Laplace transform, Diffusion and wave equations

**Fourier transforms**: Fourier integral representation, Fourier sine and cosine representation, Fourier transform pair, transform of elementary functions and Dirac delta function, finite Fourier transforms, solutions of heat, wave and Laplace equations by Fourier transforms.

**Suggested Instructional / Reading Materials:**

1. Myint UT, Partial Differential Equations for Scientists and Engineers, 3rdedition, North Holland, Amsterdam, 1987.
2. Dennis G. Zill, Michael R. Cullen, Differential equations with boundary value problems, Brooks Cole, 2008.
3. John Polking, Al Boggess, Differential Equations with Boundary Value Problems, 2nd Edition, Pearson, July 28, 2005.
4. J. Wloka, Partial Differential Equations, Cambridge University press, 1987.

**Title of the Course**: Nonlinear Waves

**Course Code:** MTH-411

**Credit Hours:** 3+0

**Course Outline**:

The continuum hypothesis and fluid particles.

Revision of transformation of partial derivatives.

Eulerian and Lagrangian descriptions of ID fluid motion.

Fluid acceleration, material derivative.

Simple problems in finding position, given velocity and vice versa.

Conservation of mass, equation of continuity.

Application to traffic flow (or something similar).

Linear solutions leading to wave equation and DAlemberts solution.

Non-linear solutions leading to kinematic wave equation and general implicit solution.

Wave steepening.

Method of characteristics.

Shocks and weak solutions: shock evolution equation.

Pressure and gravitational forces.

Conservation of momentum Eulers equation.

Ideal gas dynamics: linear and non-linear problems leading to same equations as above.

Quasi 1D flows: modified Euler equation.

Application to physiological flows, river flows and hydraulic jumps all leading to similar

equations to those already studied.

Shallow water waves, dam break problems.

Dispersion relations, wave-packets, group velocity.

Deep water gravity-capillary waves.

Finite amplitude capillary waves.

Weakly non-linear water waves the Korteweg de-Vries equation.

**Suggested Instructional / Reading Materials**

1. Supersonic Flow and Shock Waves, R. Courant & K.O. Friedrichs (Springer).
2. Linear and Nonlinear Waves, G.B. Whitham (Wiley).
3. Waves in Fluids, Sir James Lighthill (Cambridge).
4. Water Waves, J.J. Stoker (Interscience).
5. Wave Motion, J. Billingham & A.C. King (Cambridge).
6. Partial Differential Equations for Scientists and Engineers, S.J. Farlow (Dover).
7. Elementary Applied Partial Differential Equations, R. Haberman (Prentice Hall).
8. Partial Differential Equations, J. Kevorkian (Springer).
9. Applied Partial Differential Equations, Ockendon, Howison, Lacey & Movchan (Oxford)

**Title of the Course**: Dynamical Systems

**Course Code:** MTH-412

**Credit Hours:** 3+0

**Course Outline**:

Recognize the difference between a difference equation and a differential equation.

Express application problems in terms of dynamical systems equations.

Iterate functions and find orbits, fixed points, and periods.

Use graphical analysis of dynamical systems and understand phase portraits.

Understand the ramifications of the Fixed Point Theorem.

Determine the stability of fixed points (attracting vs repelling).

Recognize bifurcation points and type.

Master the ideas of the Cantor middle-thirds set.

Recognize when a dynamical system exhibits chaotic behavior (use of Sarkovskii’s

Theorem, etc.).

Generate fractals.

Recognize the common examples of fractals.

Find the topological dimension of sets and fractal dimensions.

Perform basic arithmetic of complex numbers.

Work with Julia sets and the Mandelbrot set.

identify when certain theorems apply, and if not, identify what hypothesis is violated

carry over and apply knowledge from Calculus such as differentiability and integration

prove some theorems involving dynamical systems

check results (produced both manually and with technology) and recognize those which

are obviously false or suboptimal

**Suggested Instructional / Reading Materials**

1. A First Course in Chaotic Dynamical Systems: Theory and Experiment by Robert L. Devaney

**Title of the Course**: Special Theory of Relativity

**Course Code:** MTH-413

**Credit Hours:** 3+0

**Course Outline**:

**Review:** Galilean relativity, Newtonian mechanics, Electrodynamics and inconsistency with Galilean relativity, ether and experiments for its detection, failure to detect ether. Measurement of velocity of light in moving frames. Lorentz, Poincare and developments towards relativity

**Einstein’s special theory:** Constancy of velocity of light as a postulate. Derivation of Lorentz transformation. Length contraction and time dilation. Mass- energy relation, Doppler shift. Minkowski space-time diagram, boosts as complex rotations in Minkowski space

Four dimensional space-time continuum, Lorentz transformations as coordinate transformations, vectors, scalar product, scalars, tensors, contravariant and covariant objects, laws of physics as tensor equations, Mechanics, hydro-dynamics and electrodynamics as tensor equations

**Beyond special relativity:** Inertial and gravitational mass, Equivalence principle, Introducing gravitational field as general coordinate transformation, Principle of general covariance, Metric tensor and affine connection, Gravitational potential as metric tensor, Laws of physics in presence of gravitation, gravitational time dilation and red shift, Experimental observation of gravitational red shift

**Lorentz and Poincare groups:** abelian and non-abelian groups, Rotations in two and three dimensions, generators of rotations, Representations (finite dimensional), Casimir operators, Lorentz transformations as a group, Generators for translations, rotations and boosts, Finite and infinite dimensional representations

**Suggested Instructional / Reading Materials**

1. Introduction to Special Theory of Relativity by Resnick
2. Relativity by A. Einstein
3. Classical Electrodynamics by J.D. Jackson
4. Electrodynamics by W. K. H. Panofsky & M. Phillips
5. Classical Mechanics by H. G oldstein
6. GTR and Cosmology by S. Weinberg
7. Classical Theory of Fields by L. Landau & E. Lifshitz

**Title of the Course**: Waves and Oscillations

**Course Code:** MTH-414

**Credit Hours:** 3+0

**Course Outline**:

**Mechanical Vibrations and waves:** simple harmonic motion, Mass-Spring System, Simple Harmonic Oscillator Equation, Complex Number Notation, LC Circuit, Simple Pendulum, Quality Factor, LCR Circuit

 **Forced vibrations and resonance:** Steady-State Behavior, Driven LCR Circuit, Transient Oscillator Response, Resonance, Superposition

**Coupled Oscillations:** Two Spring-Coupled Masses, Two Coupled LC Circuits, Three Spring Coupled Masses, Normal Modes, Atomic and Lattice Vibrations

**Transverse Waves:** Transverse Standing Waves, Normal Modes, General Time Evolution of a Uniform String, Phase velocity, Group Velocity.

**Longitudinal Waves:** Spring Coupled Masses, Sound Waves in an Elastic Solid, Sound Waves in an Ideal Gas.

**Traveling Waves:** Standing Waves in a Finite Continuous Medium, Traveling Waves in an Infinite Continuous Medium, Energy Conservation, Transmission Lines, Reflection and Transmission at Boundaries, Electromagnetic Waves.

**Wave Pulses:** Fourier Series and Fourier Transforms, Bandwidth, Heisenberg’s Uncertainty Principle.

**Multi-Dimensional Waves:** Plane Waves, Three-Dimensional Wave Equation, Laws of Geometric Optics, Waveguides, Cylindrical Waves.

**Interference and Diffraction of Waves:** Double-Slit Interference, Single-Slit Diffraction.

**Suggested Instructional / Reading Materials:**

1. J. Pain, “The Physics of Vibrations and Waves”, John Wiley, sixth ed. 2005.
2. P. French, “Vibrations and Waves”, CBS Publishers (2003).
3. F. S. Crawford, Jr., “Waves and Oscillations”, Berkeley Physics Course, Vol. 3, McGraw-Hill, 1968.
4. Hirose, and K. E. Lonngren, “Introduction to Wave Phenomena”, Krieger Publications, 2003.

**Title of the Course**: Fluid Mechanics-I

**Course Code:** MTH-415

**Credit Hours:** 3+0

**Course Outline**:

**Introductory Concepts**

Dimensions, units, fluid mass and weight,

Compressibility, vapor pressure, viscosity, surface tension

**Fluid Statics**

Pressure, hydrostatic force on plane and curved surface

Manometers, Plane and inclined manometers

Buoyancy and Archimedes Principle

**Elementary Fluid Dynamics**

Stream lines

Bernoulli’s Equation along the streamline and across the streamline

Application of Bernoulli’s Equation

Static, stagnation and total Pressure and pitot tube

Hydraulic grade line and energy grade line

Assumption of Bernoulli’s equation

**Fluid Kinematics**

Velocity field, acceleration field, control volume,

Material Derivative

Reynolds’s transport theorem

**Finite Control Volume Analysis**

Conservation of Mass for a Control Volume

Derivation and application of linear momentum equation

Derivation and application of momentum of momentum equation

Derivation and application of energy equation

Comparison of equations

**Differential Analysis of Fluid Flow**

Overview of types of motion and deformation a fluid element

Differential form of continuity equation

The stream function

Deriving the equations of motion

**Dimensional Analysis, Similitude, and Modeling**

Dimensional Analysis

Buckingham Pi Theorem

**Suggested Instructional / Reading Materials**

1. Fox R. W. & McDonald A. T. Introduction to Fluid Mechanics 6th ed. John Wiley & Sons, 2004.
2. White F. M. Fluid Mechanics. 5th ed. Mc Graw Hill, 2006.

**Title of the Course**: Computational Physics

**Course Code:** MTH-416

**Credit Hours:** 3+0

**Course Outline**:

**Computer Languages:** A brief introduction of the computer languages like Basic, C. Pascal etc. and known software packages of computation **Numerical Methods:** Numerical Solutions of equations, Regression and interpolation, Numerical integration and differentiation. Error analysis and

technique for elimination of systematic and random errors **Modeling & Simulations:** Conceptual models, the mathematical models, Random numbers and random walk, doing Physics with random numbers,

Computer simulation, Relationship of modeling and simulation. Some systems of interest for physicists such as Motion of Falling objects, Kepler's problems, Oscillatory motion, Many particle systems, Dynamic systems, Wave phenomena, Field of static charges and current, Diffusion, Populations

genetics etc.

**Suggested Instructional / Reading Materials**

1. ”Introduction to Computational Physics”, by M. L. De Jong, Addison Wesley, 1991 or any latest edition
2. “Applied Numerical Methods with MATLAB for Engineers and Scientists”, by S. C. Chapra, McGraw-Hill, 2nd ed. 2006 or any latest edition

**Title of the Course**: Computer Graphics

**Course Code:** MTH-417
**Credit Hours:** 3+0

**Course Outline:** Fundamentals of graphics hardware and software. Essential algorithms: line generation, solid area display, transformations, clipping, projection, Abstraction, and simple examples of libraries of graphics functions, Modelling of 3-dimensional solids, Rendering techniques and colour and lighting models, Input devices, interactive techniques, and the human-machine interface, Human factors in computer graphics.

**Suggested Instructional / Reading Materials**

Computer Graphics with Open GL, 4th edition, Hearn, Baker & Carithers.
OpenGL Programming Guide: The Official Guide to Learning OpenGL, Versions 4.3, 8th edition, Dave Shreiner,
Graham Sellers, John Kessenich, Bill Licea-Kane & The Khronos OpenGL ARB Working Group, Addison-Wesley.

**Title of the Course**: Theory of Automata

**Course Code:** MTH-418
**Credit Hours:** 3+0

**Course Outline:** Course Introduction + Fundamentals of Automata, Fundamentals of Automata + Recursive Definitions, Regular Expressions, Languages associated with regular expressions, Finite Automata, Finite Automata and their languages, Transition graphs, Transition graphs, Kleene’s theorem, Finite automata with output, Decidability, Context free grammars, Pushdown automata, CFGs and PDA, Turing machines

**Suggested Instructional / Reading Materials:**

1. “Introduction to Automata Theory, Languages, and Computation”, 3rd Edition, John E. Hopcroft, Jeffery D. Ullman and Rajeev Motwani, Pearson Education 2013.

2. “Automata, Computability and Complexity, Theory and Applications”, Elaine A. Rich, 2013.

3. “Introduction to Computer Theory”, 2nd Edition, Daniel I. A.Cohen.

**Title of the Course**: Theory of Splines

**Course Code:** MTH-419
**Credit Hours:** 3+0

**Course Outline:** The Cubic Spline: Existence, Uniqueness, and Best Approximation, Convergence, Equal Intervals, Approximate Differentiation and Integration, Curve Fitting, Approximate Solution of Differential Equations, Approximate Solution of Integral Equations, Additional Existence and Convergence Theorems. Intrinsic Properties of Cubic Splines: The minimum Norm Property, The Best Approximation Property, The Fundamental Identity, The First Integral Relation, Uniqueness, Existence, General Equations, Convergence of Lower-Order Derivatives, The Second Integral Relation, Raising the Order of Convergence, Convergence of Higher-Order Derivatives, Limits on the order of Convergence, Hilbert Space Interpretation, Convergence in Norm, Canonical Mesh Bases and Their Properties, Remainder Formulas, Transformations Defined by a Mesh, A Connection with Space Technology.

**Suggested Instructional / Reading Materials:**

Approximation Theory and Spline Functions by S. P. Singh (Editor); J. H. W. Burry (Editor); B. Watson (Editor), Call Number: 511.4 APP, ISBN: 9789400964686, Publication Date: 2011-10-19

**Title of the Course**: Optimizations

**Course Code:** MTH-420
**Credit Hours:** 3+0

**Course Outline:** Generic formulation of optimization problems, Optimization in MATLAB, Exposure to classes of optimization problems, Linear-nonlinear, continuous, constrained unconstrained, single-multiple variables, Linear programming, Nonlinear programming with constraints and no constraints, Multi-objective optimization: Pareto generation, weighted sum method, compromise programming, goal programming, heuristic optimization, Complex systems modeling: design of experiments, response surface, surrogate modeling, Optimization of practical problems.

**Suggested Instructional / Reading Materials:**

1. Optimization in Practice with MATLAB® For Engineering Students and Professionals, Achille Messac, Cambridge University Press, 2015
2. An Introduction to Optimization, 4th edition, Edwin K. P. Chong and Stanislaw H. Zak, Wiley, 2013